

# **How Does the Internet Affect the Financial Market? An Equilibrium**

## **Model of Internet-Facilitated Feedback Trading**

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# How Does the Internet Affect the Financial Market? An Equilibrium Model of Internet-Facilitated Feedback Trading\*

## Abstract

The ease of Internet stock trading has lured relatively inexperienced investors into the financial markets. We study the consequences of the influx of these uninformed traders with a dynamic equilibrium framework. Our results show that these strategic uninformed online traders who adopt feedback strategies cannot outperform those who do not follow feedback strategies and that feedback trading cannot affect market equilibrium. We also show that an informed trader's equilibrium strategy and expected profit remain unchanged with or without feedback trading. The presence of feedback trading in the market does not affect the speed at which information gets incorporated into prices. If uninformed traders aggregately adopt a more aggressive feedback trading strategy, they bear a higher risk. It is therefore important to manage and contain these uninformed traders' risks. We also discuss implications for regulating and designing such Internet trading systems.

**Keywords:** financial information, financial market, feedback strategy, market stability, online trading

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# How Does the Internet Affect the Financial Market? An Equilibrium Model of Internet-Facilitated Feedback Trading

*“I’m concerned about the great influx of new and relatively inexperienced investors who may be so seduced by the ease and speed of Internet trading that they may be trading in a way that does not match their specific goals and risk tolerances.”*

Arthur Levitt, SEC Chairman

## Introduction

Recent years have witnessed an increasing popularity of securities trading. In the financial brokerage market, the value of equities, options, and futures traded on national securities exchanges increased from \$2.23 trillion in 1990 to \$43.94 trillion in 2006.<sup>1</sup> Due to the continued growth in trading volumes, substantial investment has been made in information technology (IT) infrastructures, which led to advanced electronic trading systems (Lucas et al. 2009). These systems enable the creation of automated model-based trading and have profound implications for participants in financial markets (Clemons and Weber 1996, 1997). The fastest growth is occurring in the online trading sector (Barber and Odean 2001; Bakos et al. 2005). About 200 securities firms now offer Internet brokerage services. According to Nielsen Online, the top 10 most popular online trading sites attracted about 20 million unique visitors per month from 2007 to 2009.<sup>2</sup>

As expressed in this paper’s opening quote, SEC Chairman Arthur Levitt showed his concern that the ease and speed of Internet trading may lure relatively inexperienced investors and create problems for the market participants. This concern is not unwarranted because

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<sup>1</sup>Source: IBISWorld Industry Report on Stock Exchange in the US, December 8, 2008. <http://www.ibisworld.com/industry/default.aspx?indid=1312>.

<sup>2</sup>Source: <http://www.nielsen-online.com/>.

IT improves the information environment and reduces the cost of trading, therefore attracting participation from investors who would otherwise not participate. Various software tools empower investors and provide them with faster access to information, more complicated analysis of past prices, and simplified procedure to submit orders (Balasubramanian et al. 2003; Looney and Chatterjee 2002). The improved information and reduced cost can easily create an impression of both perceived usefulness and perceived ease-of-use. As a result, online trading systems should be associated with higher usage intentions and should lower the bar for adoption of stock trading (Davis 1989; Venkatesh et al. 2003). The availability of automated online trading for uninformed investors brings about two trends: increased stock market participation (Looney et al. 2004) and increased feedback trading (Barber and Odean 2001). Increased stock market participation can result from easier access to financial data, reduced search costs for information (Bakos 1997; Barua et al. 2004), and lower transaction costs (Stoll 2006). Feedback trading is enabled by tools in online investing systems and can be a result of investors' perceived empowerment or self-efficacy (Looney et al. 2006). In this paper, feedback trading refers to strategies based on past prices.

Under this backdrop, we examine the following research questions. First, how informed investors are affected? Specifically, if there are rational informed investors in the market who buy when prices are low and sell when prices are high (relative to fundamental values of the assets), and if each uninformed trader adopts a different feedback strategy, what is the informed traders' rational reaction? Second, how market is affected? Specifically, when the Internet enables and facilitates strategies built on past prices, what are the implications of the feedback trading on how stable the prices are (i.e., volatility<sup>3</sup>) and how fast information can be incorporated into the market? Finally, how feedback trading affect uninformed traders as a group?

In answering these questions, we model two types of investors. One type maximizes expected profits based on their superior information. The other type consists of uninformed

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<sup>3</sup>The volatility of a stock is defined as the standard deviation of percent changes of the stock price.

traders, each of whom adopts a potentially different strategy, but aggregately can be modeled as a group. Since uninformed traders do not have access to information, the orders submitted by them all depend on past price.<sup>4</sup> Our closed-form solution suggests that online trading does not increase the risk in market prices (i.e., volatility), and the equilibrium strategy and the expected profit of the informed trader remain the same with respect to different levels of feedback trading. In addition, even though the sensitivity of price in response to changes in demand (i.e., market depth) varies with respect to the intensity of aggregate feedback trading, information is incorporated into prices at a constant rate. These results offer significant implications for regulators. First, because price volatility is an important measure of market risk, our finding that price volatility does not change with increased online trading suggests that the implementation and use of online trading systems does not increase the risk for the market, even with the influx of inexperienced and uninformed traders. Second, the intensity of feedback trading does not change the equilibrium strategy and the expected profit of the informed trader. This suggests that online feedback trading does not inflict costs on informed traders and is a desirable outcome from the regulators' point of view. Finally, the speed of information getting incorporated into prices does not change with respect to the adoption of online trading systems. The stability and the liquidity of the market, two very important indicators of financial-market structure, are therefore not affected. As a result, our findings suggest that Internet-facilitated online trading will not affect the overall market, and financial regulators should not impose unnecessary limitations to the implementation and adoption of such systems.

Overall, these results suggest an optimistic outlook on the use of such online trading systems and suggest that SEC's apprehension can be somewhat relieved, at least for the market and the informed traders. To examine the implications for uninformed traders themselves, we further decompose the impact of feedback trading into two components: pure feedback

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<sup>4</sup>In the literature, the terms "momentum trader" and "uninformed trader" are often used interchangeably (Hong and Stein 1999). Momentum traders often adopt some kind of feedback strategy, which depends on past prices.

and pure noise. We find that the noise component determines the expected profit and the feedback component is associated with the variance of profit. Since the expected profit is negative, our analysis suggests that more aggressive feedback trading creates higher risks for uninformed traders without bringing higher benefits. Even worse, feedback trading creates an illusion of profit because more intensive feedback trading is associated with a higher probability of obtaining a positive profit. This result suggests that uninformed traders may bear too much risk when they only consider risk-adjusted profit (e.g., financial measures commonly calculated such as the Sharpe Ratio) when choosing strategies. For regulators, the main objective is thus to limit the risk for these uninformed traders. To this end, it is necessary to calculate and disclose risk measures (Pavlou 2003) when designing online trading systems. Uninformed traders should be encouraged to determine the stop-loss level before trading. This result echoes SEC's concern that inexperienced investors may be trading "in a way that does not match their specific goals and risk tolerances."

The rest of the paper is organized as follows. We first introduce the context of our study and review the IS-Finance literature. The next section establishes the theoretical framework. Next we examine the equilibrium property of the model and study the impact on the informed trader as well as the market. To examine the implication of feedback trading for the uninformed traders, we next compare feedback traders and noise traders and study the profits and the risks. Finally, after discussing the generalizability of the results, we conclude the paper with a discussion of this study's implications and limitations.

## **Background**

The Internet brought about many transformations to businesses. One such transformation is automated trading. Automated trading is based on advanced management information systems and has been widely used by fund managers and other institutional traders to generate and execute orders. The maturation of such information systems on the institutional

level and the reduction of transaction costs offered by discount and direct-access brokers ushered in a new era for automated trading to be available for retail investors. Retail investors are typically individual uninformed traders who rely on past prices to infer information about the true value of a security.

There are many possible explanations for why traders determine their strategies based on past prices. For example, when investors believe that there is momentum in price changes, they may adopt a trend-following strategy. In many popular investment books, trend following is deeply integrated in the proposed strategies. In a recent book (O'Shaughnessy 2006), the author motivated the book and each chapter with famous proverbs such as "I know of no way of judging of the future but by the past," and "History is a better guide than good intentions." There is a significant increase in the number of trend-following books accompanying the adoption of Internet trading (Dreman 1998; Guppy 2004; Carr 2007; Faith 2007; Coval 2009; Webb 2010). Trends are often touted as "an investor's best friends" (Lydon 2010).

Some investors believe that when the price keeps rising steadily, it will reverse and fall (O'Shaughnessy 2006). These contrarian traders buy when price drops, and short-sell when price rises. The negative feedback strategy is studied in the context of Finnish retail investors (Grinblatt and Keloharju 2000) and volatility and serial correlation in returns (Sentana and Wadhvani 1992). A recent paper shows individuals tend to buy stocks following declines in the previous month and sell following price increases (Kaniel et al. forthcoming). The authors suggest that when individual investors follow a negative feedback strategy in trading, they provide liquidity to meet institutional demand for immediacy. Bange (2000) gives empirical evidence to show that uninformed traders tend to adopt feedback trading strategies. Specifically, Bange (2000) finds that investors increase their equity holdings when they are bullish and decrease equity holdings when they are bearish. Jegadeesh and Titman (1993, 2001) empirically document that momentum strategies of buying winners and selling losers can be remarkably profitable.

Current information technologies enable them to construct various trading models and submit their orders through online trading systems. Online electronic trading systems provide the means whereby traders can process real-time price quotes, market data, and order executions quickly and at low cost (Looney et al. 2006).

For retail investors who only need trade-execution services, Internet-based online trading platforms prove to be very valuable. First, online brokerage firms, unlike traditional brokerages, typically do not have stockbrokers or branch offices. According to an industry report,<sup>5</sup> labor constitutes the largest single expense (31% of revenue) in the traditional brokerage industry. Full-service brokerages, including some investment banks, can have labor costs as high as 48% of revenue. Online brokerages therefore enjoy a very significant advantage in operation costs. Second, while most trading on traditional systems is still executed in the primary exchanges, online trading platforms typically use cost-effective alternatives, such as Electronic Communications Networks (ECNs), to match traders (Barclay et al. 2003; Kim 2007). Order routing and order execution in ECNs are central to the competition and overall efficiency of exchange markets (Fan et al. 2000). While ECNs were originally developed for brokers and institutional investors, with the development of direct-access online trading systems, retail investors can also enjoy efficiency benefits (Balasubramanian et al. 2003; Looney and Chatterjee 2002). Consequently, the cost of transaction for retail investors has significantly reduced.

Online trading systems make it significantly easier for investors to obtain past price information and establish trading strategies based on past prices. Such feedback trading and its impact on the market is the focus of this study. Both positive and negative feedback trading can be easily implemented with tools offered by online trading websites. This paper contributes to this literature by offering a theoretical model to study the impact of the use of such feedback strategies.

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<sup>5</sup>Source: IBISWorld Industry Report on Securities Brokering in the US, January 22, 2009. <http://www.ibisworld.com/industry/default.aspx?indid=1308>.



## Literature Review

Prior studies mostly focused on the incentives and implications of why brokerage firms should adopt online trading systems. We first review this literature, argue that brokers and investors are likely to be affected by the introduction of these systems, then focus on the IS studies directly related to our paper to show that a theoretical modeling perspective can contribute to our understanding of Internet-facilitated trading.

### *The Brokers*

Many companies are undertaking major initiatives to leverage the Internet to transform how they coordinate value activities. The substitution of paper, telephone, and fax with electronic transaction and information exchange based on various Internet technologies has attracted wide attention in the literature (Straub and Watson 2001). Prior works have shown that such transformations generally help firms reduce cost and improve profitability. For example, in examining how the Internet transforms operations management, Lovejoy and Whang (1995) finds that IT helps reduce inventory. Lynch and Ariely (2000) find that the Internet can significantly affect competition on price, quality, and distribution through reduced search costs. Adopting the resource-based view (Barney 1991), Barua et al. (2004) argue that firms' capabilities to coordinate and exploit IT and other resources create online informational capabilities that lead to superior operational performance. They also find that increased customer- and supplier-side digitization implies better financial performance. Several other studies of IT business value (e.g., Mata et al. 1995; Hitt and Brynjolfsson 1996; Baharadwaj 2000) point out that the complementarity between physical systems and organizational and environmental resources is most important in realizing IT value.

To capture IT value, various online trading systems were implemented. Many studies in this area focus on Electronic Communications Networks (ECNs). ECNs are electronic trading systems that can automatically match buy and sell orders without intermediaries.

Seeing ECNs as a major competitor of traditional market makers, Fan et al. (2000) call the development of such systems a “fundamental revolution.”<sup>6</sup>

Due to the importance of ECNs, many studies ensued to examine their implications to the brokerage firms and the markets. IS researchers contributed substantially to this literature. In a theoretical model, Hendershott and Mendelson (2000) study the impact of the introduction of a new trading system on market participants. They found that, contrary to the natural reactions that applaud the implementation of such systems, the effects of the new technology on market performance and investor welfare are subtle and complex. Consistent with their theoretical findings, while earlier studies identified some benefits of the implementation of ECNs, later studies find that the benefits are conditional on market contingencies. For example, Barclay et al. (1999)’s empirical study finds that market liquidity improved after the introduction of ECNs without adversely affecting market quality. Later, Barclay et al. (2003) explore the competition between ECNs and traditional market makers and find that trades are more likely to occur on ECNs when there is (1) greater information asymmetry, (2) high trading volume and (3) high stock-return volatility.

To study the introduction and adoption of new, technologically enabled business models, Eric Clemons, Bruce Weber and their colleagues develop a stream of studies that examine how existing markets become vulnerable to competition (Clemons et al. 1996, 2003; Clemons and Thatcher 2008). Clemons and Weber (1997) develop a stylized model to study how established exchanges may face different costs and risks brought about by the introduction of electronic trading systems. They propose the use of risk-based pricing to separate pricing the shares traded from pricing the services for carrying out these shares. Along the same line, Bakos et al. (2005) develop a model to study the competition between traditional and electronic brokerage firms. Using a field experiment, they find that the introduction of electronic trading firms affects the quality of service, price convergence and profits for

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<sup>6</sup>See Clemons et al. (2002) for an illustration of how ECNs were implemented in London. Looney and Chatterjee (2002) describes the web-enabled transformation of the brokerage industry.

traditional brokers. Later studies examine the entry of online trading systems more closely. For entrants, Weber (2006) describes the adoption of electronic trading at a stock exchange and suggests that about 60% of the adoption can be explained by firm-specific factors, with the remaining 40% explained by network effects. For incumbents, Lucas et al. (2009) study how IT was used in a defensive way by the New York Stock Exchange to compete with new entrants in the financial market. For the whole market, in a very recent study, Hendershott et al. (2011) evaluate the effect of algorithmic trading and conclude that algorithmic trading improves liquidity and enhances the informativeness of price quotes.

Sanjeev Dewan and Haim Mendelson develop some early theoretical models in this literature. In Dewan and Mendelson (1998), the authors show that brokers' adoption of IT trading tools can be affected by IT costs, number of traders, and the frequency and nature of new information arrivals. In a follow-up study (Dewan and Mendelson 2001), the authors integrate their model with Kyle (1985)'s and examine investments in trading tools, trading strategies and liquidity. Related to these studies, we also examine the equilibrium strategies of the market participants. However, our research topic is significantly different. In our study, we take the investment in trading tools as given. Indeed, while investment in trading tools was a significant research question in the early 2000's, fast IT penetration has rendered this issue obsolete. While Dewan and Mendelson's studies mainly offer insights on the broker side, our results have policy implications for investors.

Overall, on the brokerage side, prior studies find substantial gains in adoption of electronic trading systems and demonstrate the significant impact of such systems on brokers.

### ***The Investors***

The most important players in financial markets are unquestionably the investors. Compared to the relatively rich literature on the brokerage side, there is a lack of studies on the online trading systems' implications to investors. After the introduction of electronic trading

systems, a sufficient number of investors would be needed to trade to support a liquid market. Clemons and Weber (1996) examine the feasibility and the desirability of investors' switching to new electronic trading-based markets with a series of experiments and simulations. They find that, rather than regulatory actions favoring new markets, improved designs for IT-based trading mechanisms are needed. Individual adoption and use of IT is probably one of the most mature streams of research in IS. Prior studies focus on individual-level psychological processes and contingencies that are related to technology perceptions and situational factors (e.g., Venkatesh et al. 2003, 2007, among others). Consistent with the findings in this literature, the introduction of online trading systems was associated with increased adoption on the investor side of the market. These trading tools offer significantly more information than their traditional counterparts. As a result, the perceived empowerment and self-efficacy will lead to behavioral intention to use the system and, consequently, system use (Looney et al. 2006).

Although the empowerment of individual retail investors levels the playing field in terms of reduced cost and increased speed of order execution, IT does not turn these investors into informed or sophisticated traders. As a result, although the Internet brings a reduction in market friction, which leads to increased stock market participation, it remains uncertain how individual investors can translate improvements in access to information into superior return performance in financial markets (Bogan 2008; Looney et al. 2006). There is evidence suggesting that the performance of those traders who opt for Internet trading quickly deteriorates over time, even reaching 3% below the market (Barber and Odean 2002). In a 401(k) plan, online trades tend to have smaller portfolios and do not outperform offline traders (Choi et al. 2002). In the Korean stock market, online trading accounts for 65.3% of all stock trading in 2003, but online traders perform significantly worse than offline traders (Oh et al. 2008).

## *IS Studies of Internet-Facilitated Trading*

Table 1 summarizes IS studies in this literature that are directly related to our work of Internet-facilitated online trading. The papers are ordered chronologically to show the general transition of research topics.

Understandably, the majority of these studies are on the *broker* side since a lot of resources have been put into brokerage trading systems first. The remaining studies focusing on *investors* established necessary institutional background for our work. No theoretical work has been dedicated to study investor behavior in this literature, and we aim to fill this gap.

On the broker side, Dewan and Mendelson (1998, 2001) use a variant of Kyle (1985)'s framework to argue that IT costs, number of traders, and the frequency and nature of new information affect the level of brokers' IT investments, and that improved IT infrastructure translates into competitive advantage. This work establishes a theoretical explanation for brokers to establish e-trading systems to support retail investors' online trading. Fan et al. (2000), Hendershott and Mendelson (2000), Clemons et al. (2002), Bakos et al. (2005) Weber (2006), Lucas et al. (2009), and Hendershott et al. (2011) provide additional insights on this front. The establishment of online trading systems for retail investors offers a technical foundation for inexperienced investors to create price-based strategies. Our study contributes to this stream of literature by examining the impact on the market from the investor side.

On the investor side, Clemons and Weber (1996) study investor transition to off-exchange e-trading markets. They argue that for a new e-trading system to form a liquid and widely used market, a sufficient number of traders would need to move from traditional trading venues to it. Their experiments predict that elimination of dealer-intermediaries can diminish market quality. This work suggests that IT-based trading mechanisms should be established in existing markets and regulatory actions should not favor new markets. Clemons and Weber (1997) propose the use of risk-based pricing to preserve the existing market. A natural follow-up question is then: When the new and inexperienced investors enter the

**Table 1:** IS Studies of Online Investment

Study	Methodology	Focus	Research Question
Clemons & Weber (1996)	Experimental	Investor	Transition to e-trading system
Clemons & Weber (1997)	Simulation	Broker	E-trading system pricing
Dewan & Mendelson (1998)	Theoretical	Broker	Adoption of e-trading system
Fan et al. (2000)	Summary	Broker	Future of e-trading system
Hendershott and Mendelson (2000)	Theoretical	Broker	Introduction of new e-trading system
Dewan & Mendelson (2001)	Theoretical	Broker	Investment in e-trading tools
Clemons et al. (2002)	Summary	Broker	Impact of Internet on financial services
Looney & Chatterjee (2002)	Summary	Broker	Adoption of e-trading system
Balasubramanian et al. (2003)	Survey/Empirical	Investor	Satisfaction in online trading
Looney et al. (2004)	Survey/Empirical	Investor	Perceived online investment ability
Bakos et al. (2005)	Experimental	Broker	Entrance of e-trading systems
Looney et al. (2006)	Survey/Empirical	Investor	Computing and technical skills in e-trading
Weber (2006)	Case study	Broker	Adoption of e-trading system
Clemons & Thatcher (2008)	Summary	Broker	Competition in financial market
Lucas et al. (2009)	Case study	Broker	Adoption of e-trading system
Hendershott et al. (2011)	Empirical	Broker	Liquidity and informativeness of price

existing market, how would they change the existing strategies and market outcomes. We try to answer this question in this study.

From a behavioral point of view, Balasubramanian et al. (2003), Looney et al. (2004) and Looney et al. (2006) suggest that e-trading systems may increase the perceived ability and satisfaction of the investors and therefore may encourage their participation and induce them to take relatively more risky actions. These studies offer the psychological foundation of the feedback strategy in online trading systems.

It is unquestionable that information technology has radically changed the financial market. Internet-facilitated trading is still in its infancy, with its impact on the market far from understood. Our paper is but a small step toward understanding this important trend.

## Theoretical Framework

With some exceptions to be detailed below, our model follows closely the “noise trader” framework of Kyle (1985). In his seminal work, Kyle (1985) studies the trading strategy of an informed trader who faces noise traders. He has two main results: (1) the informed trader releases her information gradually into the market at a constant rate, and (2) the needed order volume to move the price by one dollar (i.e., market depth) is constant over time. In this paper, we consider Kyle (1985)’s results as a benchmark to study the impact of feedback trading. By comparing our results with his, we are able to assess how Internet-facilitated feedback trading may influence the market.

We consider a capital market in which trading takes place in continuous time. The market is closed and liquidated at time  $T = 1$ . One risky asset is exchanged for a riskless asset among three kinds of participants: a single risk-neutral, informed trader, who has unique access to a private observation of the *ex post* liquidation value of the risky asset;<sup>7</sup>

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<sup>7</sup>That is, the informed trader knows exactly the fundamental value of the risky asset. We use “liquidation value” and “fundamental value” interchangeably in this paper. The assumption of one single informed trader

uninformed traders, who try to infer the fundamental value of the risky asset by observing the price; and risk-neutral market makers, who set the price efficiently according to information of the quantities traded. The efficiency is achieved because the market makers are assumed to behave competitively when setting the market-clearance price.

The liquidation value of the risky asset is a realization of a random variable,  $\tilde{v}$ , which is related to the state of the real world and is assumed to follow the distribution  $N(0, \sigma_v^2)$ . All uncertainty is supported on a standard probability space  $(\Omega, \mathcal{F}, P)$ . For easy reference, Table 2 contains a table of variable definitions.

The informed trader knows both the realized value of  $\tilde{v}$  and its distribution. The market makers only know the distribution. The uninformed traders, in contrast, do not know anything about the liquidation value. At any time  $t \in [0, 1)$ , trading takes place in two steps. In step one, informed and uninformed traders place market orders by simultaneously choosing the quantities they want to trade.<sup>8</sup> In this step, the informed trader takes into consideration her private observation of the asset's liquidation value, as well as past prices and quantities she traded. She does not observe current or future prices or quantities traded by uninformed traders. Uninformed traders cannot observe the risky asset's liquidation value. Different from noise traders in Kyle (1985), the uninformed traders in this study can obtain past prices through the trading system. Based on past prices, they set algorithmic rules for the Internet-based automated system to carry out the order process.<sup>9</sup> One of the simplest types of strategic reactions to prices is positive feedback (De Long et al. 1990b). Positive-feedback

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follows from Kyle (1985). If there are multiple informed traders, information would be released faster due to competition between them. Our study focuses on how feedback trading influences the market, it is likely to have a similar impact no matter how many informed traders there are. For our main argument, it helps to keep this simple assumption. If, however the uninformed traders have some noisy signal of the liquidation value, then prior studies (e.g., Back et al. 2000; Mendelson and Tunca 2003; Pasquariello 2006, among others) show that the equilibrium outcome can be very different. In an extension and in Appendix C, we discuss this possibility and show that our results with respect to the impact of feedback trading remain unchanged.

<sup>8</sup>Market orders are typically carried out immediately. Traders decide their order volumes and the market makers choose a price to clear the market.

<sup>9</sup>We model this strategy with a predictable process. That is, each trader predefines a complete menu of responses to market conditions. The online trading system then carries out such strategies on behalf of the investors.



**Table 2:** Variable Definitions

Variable	Definition
$t$	Continuous time. The market starts at $t=0$ and clears at $t=1$ .
$\tilde{v}$	Liquidation value (true value, fundamental value) of the asset. It is normally distributed with mean zero and variance $\sigma_v^2$ .
$P_t$	Price at time $t$ .
$\mathcal{F}_t$	Available information to market makers up to time $t$ .
$\delta(t)$	A measure of the deviation of $P_t$ from the liquidation value $\tilde{v}$ up to time $t$ . Specifically, $\delta(t) = E[(\tilde{v} - P_t)^2   \mathcal{F}_t]$ . It is a very important measure of how much and how fast information gets incorporated into the prices. The smaller $\delta(t)$ is, the faster information gets incorporated into prices.
$\sigma^2$	Variance of the noise.
$W_t$	A one-dimensional standard Brownian motion. $\sigma dW_t$ indicates the noise introduced by uninformed traders.
$\sigma_v^2$	Variance of the distribution of the fundamental value $\tilde{v}$ of the asset.
$dX_U(t)$	Order submitted by the uninformed traders at time $t$ .
$dX_I(t)$	Order submitted by the informed trader at time $t$ .
$\tilde{\pi}_U(t)$	Uninformed traders' cumulative profit up to time $t$ .
$\tilde{\pi}_I(t)$	Informed trader's cumulative profit up to time $t$ .
$\beta_t$	Elasticity of feedback. It indicates how aggressive the uninformed traders' strategy is.
$\alpha_t$	Informed trader's aggressiveness in adjusting in response to the gap between price and fundamental value.
$\lambda_t$	The increase in price as a result of one unit of increase in total demand at time $t$ . It is a measure of how sensitive the market price is with respect to demand. It is related to the concept of market depth, which is defined as $1/\lambda_t$ .

investors buy securities when prices rise and sell when prices fall. In De Long et al. (1990b), all uninformed traders follow a positive-feedback strategy. In our model, we do not limit our attention to positive-feedback investors; instead, a strategic uninformed trader may take any possible strategy with respect to price, and we examine their aggregate effect on the market.

In step two, market makers set the price according to aggregate orders and clear the market. For example, if there are more buy-orders than sell-orders, the market makers will increase the price to a certain level, then take a position to sell the asset to the buyers. When doing so, their information consists of observations of current and past aggregate quantities traded by informed and uninformed traders. Market makers do not know the liquidation value, nor do they know the identity of the traders (informed or uninformed) who submit the orders. As a result, price fluctuations are jointly determined by the changes in order flow. Following Kyle (1985), all orders in our paper are market orders. When submitting orders in step one, the investors only need to specify the trading volumes (i.e.,  $dX_I(t)$  and  $dX_U(t)$ ). When the market makers receive these orders, they increase (decrease) the price if there are more (fewer) buy orders than sell orders, so that the price can reflect all available information.<sup>10</sup> Similar to Kyle (1985), we are not specifically interested in the strategies of the market makers, so they do not explicitly maximize any particular objective. We assume a competitive segment of market makers so that all available information in the market can be incorporated into prices through the competition of these market makers.

Each uninformed trader may set a different trading rule that reflects different reactions to price changes. Among the  $m$  uninformed traders, trader  $i$ 's order at time  $t$  can be written as

$$dX_U^i(t) = \beta_t^i dP_t + \sigma_i dW_t^i, \quad (1)$$

where  $(W_t^i, i = 1, 2, \dots, m)$  is an  $m$ -dimensional Brownian motion defined on  $(\Omega, \mathcal{F}, P)$ .  $\sigma_i$  is a scaling factor that describes the magnitude of noise trading, and only depends on

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<sup>10</sup>Equation (7) below specifies how the price is adjusted.

individual  $i$ .  $\beta_t^i$  is the feedback elasticity parameter at time  $t$  that reflects the reaction to the price process up to time  $t-$ .<sup>11</sup> Throughout this paper, the notation  $t-$  means the time just before  $t$ .  $P_t$  is the price process of the risky asset, chosen by the market maker in reaction to total demand at time  $t$ . It is chosen by the market makers at time  $t$  after observing the aggregate orders. The term  $dP_t$  indicates the price change immediately preceding time  $t$ . Since uninformed traders do not have information about liquidation value,  $\tilde{v}$  and  $(W_t^i)$  are independent. In this paper, we put no restrictions on the possible strategies the individuals may adopt (positive or negative feedback); thus depending on the individual strategies,  $\beta_t^i$  can be positive or negative, can change over time, and can even be a function of past prices.<sup>12</sup> A greater  $|\beta_t^i|$  means a more proactive approach in trader  $i$ 's feedback strategy. In general, for every unit change in recent price, a trader with a greater  $|\beta_t^i|$  would submit a larger buy or sell order, depending on the sign of  $\beta_t^i$ . We do not require all uninformed traders to follow the same Brownian motion when introducing noise.

In this paper,  $\beta_t^i$  describes the strategy taken by each uninformed trader and measures how each trader reacts to price changes. In addition to the benefit of simplicity, this formulation has a couple of additional features. First, it reflects the fact that the uninformed traders can only setup such a rule through the Internet trading system. More fine-tuned strategies have to be implemented with more human interference. Since we would like to focus on the effects of Internet-facilitated feedback trading, we do not consider such human activities. Second, this assumption actually allows a level of flexibility in investors' strategies. As uninformed traders are relatively naïve investors in our model, there may be a large variety of possible strategies for them to follow and they may change their strategies

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<sup>11</sup>Strictly speaking,  $\beta_t^i$  is a predictable stochastic process determined by individual  $i$  who accounts for the information available from observing the price process up to time  $t-$ . This is fully consistent with the use of IT in online trading systems, in which the users set predefined strategies in their trading algorithms. Note that such strategies can be quite elaborated. For example, a potential strategy can specify that even if the price level is the same the feedback elasticity is different when the price change is different. Alternatively, when price change is the same, the feedback elasticity can be different when the price is at different levels. The bottom line is that  $\beta_t^i$  is capable to accommodate different strategies that depend on past prices and price changes.

<sup>12</sup>The reason that  $\beta_t^i$  may change is that traders may have different interpretations about  $dP_t$  when  $P_t$  and the general market situations are different.

frequently. As long as their strategies are based on some rule that responds to price changes, our framework yields valid equilibrium depiction of the market. For example, some  $\beta_t^i$  may be a function of past prices or other variables, our results would remain the same because we do not impose any limitations on how each individual chooses the value of this parameter. As demonstrated in the literature, user heterogeneity often play a significant role in affecting the use of information systems (Davis 1989; Thatcher and Perrewé 2002; Bapna et al. 2004), our formulation allows a wide range of values of  $\beta_t^i$  to be heterogeneous. Consequently, the results are quite general with respect to all possible feedback strategies that the traders may adopt. In addition, this simple formulation enables us to conduct comparative statics after we solve the model in closed form.

Since the order flow of the uninformed traders cannot be distinguished on the individual level by the market makers, we can use a representative agent to discuss the aggregate order submitted by uninformed traders. This treatment turns out to simplify our analysis significantly.

Define  $X_U(t) = \sum_{i=1}^m X_U^i(t)$ ,  $\beta_t = \sum_{i=1}^m \beta_t^i$ , we have

$$dX_U(t) = \beta_t dP_t + \sum_{i=1}^m \sigma_i dW_t^i. \quad (2)$$

After normalization,  $\sum_{i=1}^m \sigma_i dW_t^i$  can be written as  $\sigma dW_t$ , where  $W_t$  is a one-dimensional standard Brownian motion. Equation (2) can thus be conveniently rewritten as

$$dX_U(t) = \beta_t dP_t + \sigma dW_t. \quad (3)$$

The first term of the right-hand side of (3) gives a simple formulation of how uninformed traders react to past price. Again,  $\beta_t$  is a predictable stochastic process that contains all information up to time  $t-$ . This formulation is similar to De Long et al. (1990a). Different from their study, we do not assume  $\beta_t$  to be positive at all times. When the process  $\beta_t$

is positive at time  $t$ , uninformed traders play the positive feedback strategy aggregately. Depending on the price process and the rate of change in price, each uninformed trader adopts a strategy, and aggregately, their order in response to  $dP_t$  at time  $t$  is reflected by  $\beta_t$ .

We choose this form based on the consideration that (1) this formulation captures the simplest form of rational decisions (in the spirit of using past prices as a source of information) made by uninformed traders; (2) the formulation's simplicity enables us to derive closed-form solutions that offer insights into the interactions between informed and uninformed traders; and (3) at the aggregate level, as long as uninformed traders are strategic, or in other words, when they use past price to infer information, their reaction to changes in price should not always be zero. In our model, if  $\beta_t = 0$  for all  $t$ , then it is equivalent to say that the uninformed traders as a group do not care about past price changes at all. If  $\beta_t$  is zero for all  $t$ , uninformed traders degenerate into the noise traders of Kyle (1985). Because there are many uninformed traders in the market, and they have different reactions to the price process,  $\beta_t$  cannot be a constant either. We see  $\beta_t$  as a measure of how aggressive uninformed traders are when they establish trading rules for the automated trading system. The second term on the right hand side of (3) reflects the noise trading introduced by uninformed traders.<sup>13</sup>

The quantity traded by the informed trader at time  $t$  is denoted  $dX_I(t)$ ; then the profit of the informed trader, denoted  $\tilde{\pi}(t)$ , is given by

$$d\tilde{\pi}(t) \equiv d\tilde{\pi}(X_I(t), P_t) = (\tilde{v} - P_t)dX_I(t). \quad (4)$$

An equilibrium in this market consists of trading strategies  $\{X_I(t)\}_{0 < t < 1}$  for the informed, and a pricing rule  $\{P_t\}_{0 < t < 1}$  for the market makers, such that two conditions are met:

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<sup>13</sup>An implicit assumption here is that there is no bound on the total amount of the security that can be traded in the market. This assumption ensures that noise traders do not substantially affect the price in the sense of violating the semi-strong efficiency condition. It is a standard assumption in the literature. For example, in his discrete model, Kyle (1985) assumes a normal distribution with mean zero and variance  $\sigma^2$ . In a dynamic model with continuous time similar to ours, Back (1992) assumes an "unbounded" Brownian motion (in his paper, the process  $Z_t$  is a Brownian motion independent of  $\tilde{v}$ , with zero mean and variance  $\sigma^2$ ). In reality, this assumption is likely to be satisfied because each order is usually relatively small compared to the market value of the securities.

(1) Profit Maximization: For any other trading strategy of the informed,  $\{Y_I(t)\}_{0 < t < 1}$ ,

$$\tilde{\pi}(X_I(t), P_t) \geq \tilde{\pi}(Y_I(t), P_t);$$

(2) Semi-Strong Market Efficiency: The random variable  $P_t$  satisfies

$$E(\tilde{v}|\mathcal{F}_t) = P_t,^{14} \tag{5}$$

where  $\{\mathcal{F}_t\}_{0 < t < 1}$  is the natural filtration generated by the aggregate order process  $\{X_I(t) + X_U(t)\}_{t \in (0,1)}$ . That is, the price reflects the information available to market makers up to time  $t$ . The market efficiency condition is obtained through our assumption of competitive market makers. At time  $t$ , the market makers receive the market order from the investors,  $dX_U(t) + dX_I(t)$ , which is based on all information up to time  $t-$ . The market makers first set a new price  $P_t$ , which is higher (lower) than  $P_{t-}$  if there are more buy-orders (sell-orders). The market is cleared by market makers taking the position of  $-(dX_U(t) + dX_I(t))$ . Their profit from this transaction is  $-(\tilde{v} - P_t)[dX_U(t) + dX_I(t)]$ . To obtain zero expected profit, they have to take expectations based on information available to them by time  $t$  to get

$$E[\tilde{v} - P_t|\mathcal{F}_t] = 0,$$

which is equivalent to (5).

In equilibrium, the choice of the uninformed traders,  $\beta_t$ , can potentially have an impact on the price process through the orders and then affect the strategy of the informed trader.<sup>15</sup>

After seeing an updated price, the informed trader adjusts her order according to the difference between the current price,  $P_t$ , and her private observation of the liquidation value,

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<sup>14</sup>Here and throughout the paper the symbol  $E$  denotes expectation taken over the Brownian motion and  $\tilde{v}$ . We explicitly indicate known information with the notation of conditional expectations when it is appropriate.

<sup>15</sup>As we show below, however, the uninformed traders' strategy does not affect the price process.

$\tilde{v}$ . Following the literature (Kyle 1985; De Long et al. 1990b), we can write,

$$dX_I(t) = \alpha_t(\tilde{v} - P_t)dt, \quad (6)$$

and the pricing rule of the market makers is described by

$$dP_t = \lambda_t(dX_I(t) + dX_U(t)), \quad (7)$$

where  $\alpha_t$  and  $\lambda_t$  are strictly positive functions to be determined in the equilibrium.<sup>16</sup>

Equation (6) describes the informed trader's strategy. Given the difference between the current price,  $P_t$ , and the liquidation value,  $\tilde{v}$ , the informed trader chooses  $\alpha_t$  to decide the order to be submitted at time  $t$ ,  $dX_I(t)$ . Her profit maximization problem is

$$\max_{\alpha_t} E \left[ \int_0^1 (\tilde{v} - P_t) dX_I(t) \middle| \tilde{v} \right]. \quad (8)$$

Equation (7) models the market makers' response to total demand. It is through this process, information gets incorporated into price. For example, if there are more buy-orders than sell-orders, the overall order is positive.  $dP_t$  will be positive because  $\lambda_t$  is strictly positive. Each unit of increase in total demand will increase the price by  $\lambda_t$ , which will be endogenously determined in equilibrium.

As in Kyle (1985), market makers do not explicitly maximize any particular objective. In this sense, Equation (7) is only a description of how market orders change price. We need to write this equation out explicitly in order to examine how changing  $\alpha_t$  affects  $\lambda_t$ .

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<sup>16</sup>In this paper,  $\beta_t$  is a predictable process, so it appears in all functions. When the informed trader chooses the optimal  $\alpha_t$ , it is potentially a function of  $\beta_t$ . We can write  $\alpha_t \equiv \alpha_t(\beta_t, \theta)$ , where  $\theta$  is a vector of other exogenous variables such as  $\sigma$ ,  $\sigma_v$  and  $t$ . Although  $\lambda_t$  is not a decision variable (in the sense that market makers do not optimize any objectives), it is endogenously determined. So  $\lambda_t$  is a function of other variables. We can write  $\lambda_t \equiv \lambda_t(\alpha_t, \beta_t, \theta)$ . In the following, for notational simplicity, we do not write them as functions.

To examine how much and how fast information gets incorporated into prices, we define

$$\delta(t) = E \left[ (\tilde{v} - P_t)^2 \middle| \mathcal{F}_t \right] \quad (9)$$

as a measure of the deviation of the price at time  $t$ ,  $P_t$ , from the liquidation value,  $\tilde{v}$ . At a certain time  $t$ , the smaller the value of  $\delta(t)$ , the faster information gets incorporated into prices.

The trading strategy  $dX_I(t)$  and the pricing rule  $dP_t$  are characterized by the positive functions  $\alpha_t$  and  $\lambda_t$ , respectively. A greater  $\alpha_t$  is associated with a more radical adjustment in response to the gap between price and liquidation value. The informed trader will choose this  $\alpha_t$  optimally to maximize her profit. The parameter  $\lambda_t$  gives the change in price as a result of one unit increase in total demand at time  $t$ . It measures the sensitivity of financial security price to demand and is often called market depth.<sup>17</sup> In our model,  $\lambda_t$  is jointly decided by other parameters in the equilibrium.

## Properties of Equilibrium

In this section, we examine the equilibrium outcome when uninformed online traders strategically respond to price according to (3). We examine the price process, the informed trader's optimal strategy, and feedback trading strategy's impact on market depth.

### *The Equilibrium*

To derive equilibrium conditions, our objective is to characterize the relations among  $\alpha_t$ , the optimal strategy of the informed;  $\lambda_t$ , a measure of market depth; and  $\beta_t$ , uninformed traders' aggregate feedback trading strategy.

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<sup>17</sup>Formally, market depth is defined as  $\frac{1}{\lambda_t}$ .



**Proposition 1.** *The linear equilibrium is characterized by (3), (6), and (7), with*

$$\alpha_t = \frac{\lambda_t \sigma^2}{\left(\sigma_v^2 - \int_0^t \left(\frac{\lambda_s \sigma}{1 - \lambda_s \beta_s}\right)^2 ds\right) (1 - \lambda_t \beta_t)}, \quad (10)$$

$$\frac{\sigma^2}{\alpha_t} \left[ \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right] = \frac{1}{\lambda_t} - \beta_t, \quad (11)$$

$$\frac{1}{\lambda_t} - \beta_t > 0, \quad (12)$$

and in the meantime,  $\lambda_t$  satisfies that for all  $t \in [0, 1)$ ,

$$\sigma_v^2 - \sigma^2 \int_0^t \left( \frac{\lambda_s}{1 - \lambda_s \beta_s} \right)^2 ds > 0. \quad (13)$$

**Proof:** All proofs are in Appendix A.

Proposition 1's importance lies in the characterization of the relation among  $\beta_t$ ,  $\alpha_t$  and  $\lambda_t$ . If the market is viewed as a black box with the input of  $\beta_t$ , then these relations give the internal mechanisms in which the endogenous variables need to satisfy in order for the equilibrium to hold. Equation (10) represents the informed trader's strategy in terms of  $\lambda_t$  and  $\beta_t$ . Since  $\alpha_t$  in Equation (11) is always positive, Equation (12) gives an upper bound to  $\lambda_t$ . Given (12), the other term in the denominator of the right hand side of (10) should also be positive, thus we have (13).

We are now ready to characterize the equilibrium behavior of the informed trader. Our objective is to find out whether or to what extent online feedback trading affects the equilibrium strategy of the informed trader. The opening quote of the paper reveals the policy makers' concerns about the influx of uninformed traders. When these inexperienced traders enter the market and follow a feedback strategy that is easily enabled by automated trading systems, it is certainly important to examine the risks faced by these uninformed traders. A more crucial issue to examine, however, is if the presence of the uninformed traders affect the market in any way negatively. We thus examine how feedback trading may change the

strategy of the informed trader.

Given her information about  $\tilde{v}$ , the informed trader would choose  $\alpha_t$  that maximizes her profit. That is,

$$\begin{aligned}
\max_{\alpha_t} E \left[ \int_0^1 (\tilde{v} - P_t) dX_I(t) \middle| \tilde{v} \right] &= \max_{\alpha_t} E \left[ \int_0^1 (\tilde{v} - P_t) \alpha_t (\tilde{v} - P_t) dt \middle| \tilde{v} \right] \\
&= \max_{\alpha_t} \int_0^1 \alpha_t E \left[ E \left[ (\tilde{v} - P_t)^2 \middle| \mathcal{F}_t \right] \right] dt \\
&= \max_{\alpha_t} \int_0^1 \alpha_t E[\delta(t)] dt \\
&= \max_{\alpha_t} \int_0^1 \alpha_t \left( \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right)^{-1} dt, \tag{14}
\end{aligned}$$

subject to (13), where the third equality follows the definition of  $\delta(t)$  (i.e., Equation 9), and the last equality is due to Equation (25) in the proof of Proposition 1.

In Proposition 1, the actions taken by the informed trader to maximize her expected profit (by choosing an optimal  $\alpha_t$ ) will have an impact on the sensitivity of price to demand (i.e.,  $\lambda_t$ ).

Using the result of Equation (11), when  $\alpha_t \left( \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right)^{-1}$  is maximized,  $\sigma^2 \left( \frac{1}{\lambda_t(\alpha_t)} - \beta_t \right)^{-1}$  is also maximized. Here we explicitly write out  $\lambda_t$  as a function of  $\alpha_t$  to indicate that the informed trader's optimization leads to changes in  $\lambda_t$ .

We can then rewrite the maximization problem in Equation (14) to

$$\max_{\alpha_s} \int_0^1 \frac{\lambda_s(\alpha_s) \sigma^2}{1 - \lambda_s(\alpha_s) \beta_s} ds, \tag{15}$$

subject to (13).

Summarizing the above argument, we can transform the maximization problem of the informed trader and write the following Proposition.

**Proposition 2.** *The informed trader would choose  $\alpha_t$  such that  $\int_0^1 \frac{\lambda_t(\alpha_t)}{1 - \lambda_t(\alpha_t) \beta_t} dt$  is max-*

imized subject to  $\sigma_v^2 - \int_0^t \left( \frac{\lambda_s(\alpha_s)\sigma}{1 - \lambda_s(\alpha_s)\beta_s} \right)^2 ds > 0$  for all  $t \in [0, 1]$ . When and only when (15) is maximized,  $\frac{\lambda_t}{1 - \lambda_t\beta_t} = \frac{\sigma_v}{\sigma}$ .

Proposition 2 establishes that market depth  $1/\lambda_t$  is determined by three factors: (1) the trading strategy of the uninformed traders  $\beta_t$ , (2) the variance of the liquidation value  $\sigma_v$ , and (3) the level of noise trading  $\sigma$ .

Kyle (1985)'s model suggests that  $\lambda_t = \frac{\sigma_v}{\sigma}$ , which is a special case of our result, with  $\beta_t = 0$ . It is clear that when uninformed traders aggregately play feedback strategies, they have a non-negligible influence on how sensitive the market price is to demand. A deeper market would need a larger order to change the price by one unit. While the market depth is a constant in Kyle's study, it is no longer so in our model when we consider feedback traders. Proposition 2 suggests that the market depth is constantly changing with respect to the magnitude of feedback trading. The stronger the feedback, the smaller  $\lambda_t$  is, thus the deeper the market. This result implies that when feedback trading is increased, the price becomes less responsive to orders. We use the following theorem to summarize the results so far and formally show the determinants of market depth, informed trader's strategy, risks in market price (i.e., volatility) and informed trader's profit.

**Theorem 3.** *The linear equilibrium is characterized by (3), (6) and (7), with*

$$\lambda_t = \frac{\sigma_v}{\sigma + \sigma_v\beta_t}, \quad (16)$$

$$\alpha_t = \frac{\sigma}{\sigma_v(1 - t)}. \quad (17)$$

*The deviation of the price from the liquidation value at time  $t$  is*

$$\delta(t) = \sigma_v^2(1 - t). \quad (18)$$

The informed trader's expected profit at time  $t$  is given by

$$E(\tilde{\pi}_I(t)) = \sigma_v \sigma t. \quad (19)$$

Consistent with Proposition 2, Equation (16) suggests that the needed order size to move the market price by one dollar is no longer a constant when there are feedback traders. The stronger the feedback, the more difficult it is for the orders to move the price. This result is intuitive. The orders from uninformed traders are not based on new information, but they make it more difficult for the market makers to adjust the price to reflect public information. While  $\lambda_t$  in Kyle (1985) is a constant, we show that market becomes deeper (more difficult to move the price by one dollar) when feedback trading is stronger. In the market,  $\beta_t$  determines  $\lambda_t$ , but the relation also imposes a restriction on  $\beta_t$ . Note that  $\lambda_t$  is positive, so the denominator of the right hand side of (16) should also be positive. This suggests that  $\beta_t > -\frac{\sigma}{\sigma_v}$ . For the semi-strong efficient market condition to hold, it is also required that  $\beta_t < \alpha_t$ .<sup>18</sup> Overall, in our paper, the equilibrium conditions require a bound on the strength of feedback:  $\beta_t \in \left(-\frac{\sigma}{\sigma_v}, \alpha_t\right)$ . This bound suggests that the efficient market hypothesis is restrictive even though it is a powerful tool.

Note that market depth is a function of feedback elasticity and is not a constant. We argue that constant market depth is not a necessary condition for information to be incorporated into prices at a constant rate. This can be understood from (7), the pricing rule:

$$dP_t = \lambda_t(dX_I(t) + dX_U(t)).$$

When uninformed traders only bring in noise, the term  $dX_U(t)$  does not introduce any new information. A constant  $\lambda_t$  implies that the private information gets incorporated

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<sup>18</sup>De Long et al. (1990b) argue that if  $\beta_t \geq \alpha_t$  then the market price will not converge to the liquidation value at the time of market clearance (in our setting, when  $t = 1$ ).

into price at a constant rate. When uninformed traders adopt feedback strategies, they actually magnify the information introduced by  $dX_I(t)$ . For one unit of change in  $X_I(t_1)$ , there will be a certain unit of change in  $X_U(t_2)$  (with  $t_1 < t_2$ ) that contains essentially the same information. Since the informed trader's strategy (i.e.,  $dX_I(t_1)$ ) does not change in the presence of feedback traders, and the price process (i.e.,  $P_t$ ) does not change, the non-constant market depth plays the very role of keeping a constant rate of incorporating information into prices. Equation (16) suggests that in equilibrium market depth completely absorbs the impact of feedback trading. This can be seen from (18), in which  $\delta(t)$  is a linear function of  $t$ . Since  $\delta(t)$  is not affected by  $\beta_t$ , it is independent from the magnitude of feedback trading. Therefore, the speed information gets incorporated into prices is constant and does not vary with feedback trading.

Theorem 3 shows that the aggregate impact introduced by uninformed traders gets completely absorbed by market depth. For the informed trader, the optimal trading strategy and the expected profit remain unchanged. Moreover, feedback trading has no impact on price volatility. Note that (17), (18), and (19) are the same as in Kyle (1985). Comparing Theorem 3 with the results in Kyle (1985) suggests that more aggressive feedback trading does not affect informed trader's strategies.

On the empirical ground, our result is highly consistent with prior findings (Barber and Odean 2002; Choi et al. 2002). These studies show that while the use of the Internet is associated with an increase in the frequency of trading, it has an insignificant impact on the trading volume and the price levels.

Our results offer opportunities to conduct empirical research on related issues. For example, a testable hypothesis directly following from this study is that feedback trading brings no impact on the market and on the informed trader's strategy. This can be verified in an experimental setting. Market simulation tools would make the design of such experiments straightforward (Schwartz et al. 2006; Bloomfield and Anderson 2010). Based on Kyle (1985)'s model, Ellison and Mullin (2008) derive a structural empirical framework to study

the speed of information getting incorporated into financial security prices with secondary data. Their empirical model can be directly adapted to study changes in market depth. An empirical challenge is to find the right setting because we need an exogenous shock that changes investors' access to such tools in order to examine the impact of feedback trading on the market. When more and more Internet-facilitated trading systems are deployed in various markets, we believe empirical research in this area is bound to be fruitful.

## **Is Feedback Trading Preferable to Noise Trading?**

Now that we have established the equilibrium of the market and found that feedback trading has no significant impact on either the informed traders or the price process, naturally it is interesting to study the profit and risk of the uninformed traders.

### ***Expected Profit of Uninformed Traders***

In models of noise trading, noise traders are an indispensable ingredient for the existence of financial markets. Whenever an exchange takes place, either the buyer or the seller makes a mistake. If the market is solely composed of informed traders, no trading can take place.<sup>19</sup> Black (1986) argues: "With a lot of noise traders in the market, it now pays for those with information to trade. ... Most of the time, the noise traders as a group will lose money by trading, while the information traders as a group will make money." Kyle (1985, p.1330) also finds that the informed trader's expected profits equal noise traders' expected losses.

Black (1986)'s explanation is that noise traders do not know they are trading on noise, and even information traders are not sure whether they are trading on real information or merely noise. De Long et al. (1990a; 1991), and recently Hirshleifer et al. (2006) identify conditions under which noise traders can earn higher expected returns than informed traders

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<sup>19</sup>This is assuming, for simplicity, that all informed traders have the same information. See Theorem 7 for an extension to relax it.

and conditions under which uninformed traders not only survive, but also dominate the market.

From the derivation of the results so far, it is not difficult to calculate the expected profit of the uninformed traders. We formalize the finding with a proposition.<sup>20</sup>

**Proposition 4.** *In equilibrium, the profit earned by uninformed traders from time 0 to t is*

$$E \left[ (1-) \int_0^t (\tilde{v} - P_t) dX_U(t) \right] = -\sigma\sigma_v t$$

This result does not depend on the value of  $\beta_t$ . This implies that in our model, when the uninformed traders adopt more aggressive feedback trading they do not gain higher profits.

Kyle (1985) states that the uninformed traders' loss is exactly the gain of the informed. Proposition 4 suggests that even if uninformed players use a feedback strategy with the help of online trading tools, their profit remains the same. Note that if we set all  $\beta_t = 0$  in the proof of Proposition 4, we also give a proof for the zero-sum statement made in Kyle (1985).<sup>21</sup> This result echoes our discussion in the introduction: Although automated trading empowers uninformed traders in terms of lower search and transaction costs and an increased speed of order execution, it does not offer any information advantage.<sup>22</sup> Our result here gives theoretical support to suggest that online feedback traders should not outperform their offline counterparts in terms of expected profit.

### ***Variance of Uninformed Traders' Profit***

We have shown that the expected profit of feedback traders does not vary with the magnitude of feedback. How about the risks taken by the traders when they adopt different levels of

<sup>20</sup>In this proposition, “(1-)  $\int$ ” denotes the stochastic integral that takes the right endpoint in the Riemann sum. The rationale behind this is that the price is formed *after* the submission of market orders.

<sup>21</sup>Such a proof is missing in Kyle (1985), and it is not trivial.

<sup>22</sup>There are a number of studies in the literature examining the impact of the use of decision aids (e.g., Todd and Benbasat 1992). Consistent with the literature, we find that the mere use of decision aids does not yield an information advantage.

feedback strategies? It is important to study the variance of the profits, because  $\beta_t$  reflects how aggressive the automated trading strategies are. The purpose of this part is to compare the variance of the expected profits of the two types of traders (noise vs feedback).

We can start from the price process in equilibrium,

$$dP_t = \frac{1}{1-t}(\tilde{v} - P_t)dt + \sigma_v dW_t.$$

The solution to this stochastic differential equation is,

$$P_t = P_0(1-t) + \tilde{v}t + (1-t) \int_0^t \frac{\sigma_v}{1-s} dW_s,$$

where  $P_0$  is the initial value of price  $P_t$ .

We relegate the mathematical derivation to Appendix A and give the following theorem.

**Theorem 5.** *Let  $\beta$  denote the mean feedback intensity over the whole period. Let  $D(\tilde{\pi}_I)$  and  $D(\tilde{\pi}_U)$  denote the variances of the profits for the informed trader and uninformed traders up to time  $T = 1$ , respectively. We have*

$$D(\tilde{\pi}_I) = \frac{1}{4}\sigma_v^2\sigma^2,$$

and

$$D(\tilde{\pi}_U) = \sigma_v^4\beta^2 + \frac{1}{2}\sigma^2\sigma_v^2.$$

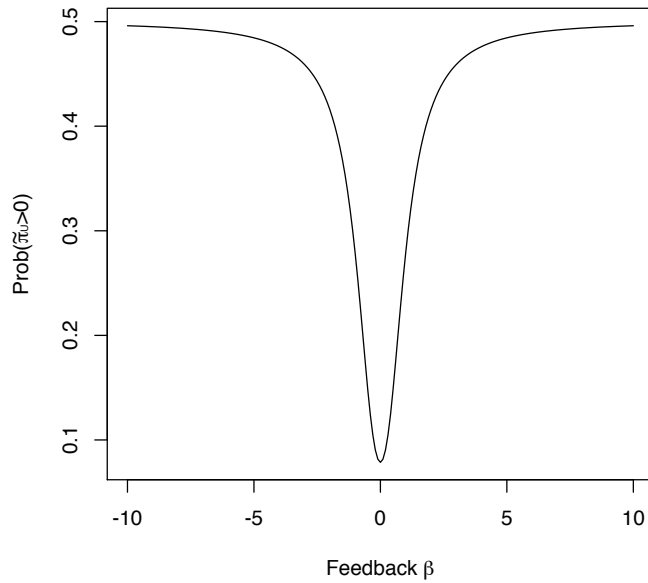
For noise traders, when  $\beta = 0$ ,

$$D_N(\tilde{\pi}_U) = \frac{\sigma_v^2\sigma^2}{2}, \tag{20}$$

where  $D_N(\tilde{\pi}_U)$  denotes the variance of profit for *pure* noise traders.



Equation (20) is a very intuitive result. It adds insight to Kyle's (1985) noise trader discussion. When uninformed traders are all noise traders, this result indicates that the variance of the profit is decreasing in the precision of the signal and increasing in the level of noise trading. When  $\beta \neq 0$ ,  $D(\tilde{\pi}_U) = \sigma_v^4 \beta^2 + \frac{1}{2} \sigma^2 \sigma_v^2 > \frac{1}{2} \sigma^2 \sigma_v^2 = D_N(\tilde{\pi}_U)$ . The greater the absolute value of  $\beta$ , the greater the variance of the expected profit. Naturally, investors who are more risk-loving would want to choose higher values of  $\beta$  in order to have a chance for a very high profit.<sup>23</sup>



**Figure 1:** Probability of Obtaining Positive Profit.

Given that uninformed traders know they earn a negative expected profit, it pays for them to increase the variance of the profit, as this allows a higher probability to obtain a positive profit (even though the expected loss remains the same). Figure 1 demonstrates why a more aggressive feedback strategy is preferred if the investor tries to increase the chance of getting a positive profit (for expositional simplicity, assuming a normal distribution, and without generality, setting  $\sigma_v = \sigma = 1$ ). The x-axis is the level of feedback trading, and

<sup>23</sup>In the next section, we examine the illusion of risk-adjusted profit of investors.

the y-axis shows the probability of uninformed traders obtaining a positive profit. When a certain investor reduces her absolute value of  $\beta$ , she reduces her chance to obtain positive profit. In fact, the probability of positive profit ( $\text{Prob}(\tilde{\pi}_U > 0|\beta)$ ) is the lowest at  $\beta = 0$ .

### *The Illusion of Risk-Adjusted Profit*

In previous sections, we have studied the impact of feedback trading on the market. We now turn to examine the expected profits of uninformed investors who adopt different particular strategies. We achieve this goal by separating the effects of feedback trading from those of pure noise trading.

The financial industry often use Sharpe Ratio to calculate risk-adjusted profits of securities. If we write down  $\pi_U^{adj} \equiv \frac{\tilde{\pi}_U}{\sqrt{D(\tilde{\pi}_U)}}$  as the risk-adjusted profit for the uninformed trader, where the subscript  $U$  can be  $N$  (Noise, for  $\beta = 0$ ) or  $F$  (Feedback, for  $\beta \neq 0$ ), then we always have  $\pi_F^{adj} > \pi_N^{adj}$ . We also have that, for feedback traders whose  $\beta \neq 0$ , for all  $|\beta_1| \geq |\beta_2|$ ,  $\pi_{F\beta_1}^{adj} \geq \pi_{F\beta_2}^{adj}$ . If interpreted in the conventional way, these results seemingly suggest that the risk-adjusted profit from feedback trading is greater than that from pure noise trading. We next show this is an illusion.

In Equation (3), when uninformed traders submit their market orders, they also introduce noise (the term  $\sigma dW_t$ ) into the market. To isolate the market impacts of feedback trading and noise trading, we write the demand out of pure feedback as  $dX_F(t) = \beta_t dP_t$  and the demand out of pure noise as  $dX_N(t) = \sigma_t dW_t$ . Note that pure feedback trading is impossible in the model, we consider these two components separately for theoretical reasons. From the derivation of Proposition 5, we immediately have,

**Proposition 6.** *In equilibrium, the profit earned by pure feedback trading from time 0 to  $t$ ,  $t \in [0, 1]$ , is*

$$E \left[ (1-) \int_0^t (\tilde{v} - P_t) dX_F(t) \right] = 0$$

and its variance is

$$D(\tilde{\pi}_F) = \sigma_v^4 \beta^2.$$

Proposition 6 suggests that the expected profit and the variance of profit we derived in previous sections for uninformed traders can both be decomposed into two parts. One part comes from feedback trading and the other part from noise trading.

**Table 3:** Decomposition of Mean and Variance of Profit for Uninformed Traders

	Expected Profit	Variance of Profit	Risk-Adjusted Profit
Pure Feedback Trading (P)	0	$\sigma_v^4 \beta^2$	0
Noise Trading (N)	$-\sigma \sigma_v$	$\frac{1}{2} \sigma^2 \sigma_v^2$	$-\frac{1}{\sqrt{\frac{1}{2}}}$
Noisy Feedback Trading (i.e., Uninformed Trading) (U)	$-\sigma \sigma_v$	$\sigma_v^4 \beta^2 + \frac{1}{2} \sigma^2 \sigma_v^2$	$-\frac{1}{\sqrt{\frac{\sigma_v^2}{\sigma^2} \beta^2 + \frac{1}{2}}}$

Table 3 shows how the decomposition works. Uninformed traders as defined in previous sections (or noisy feedback traders) can take two roles in the market. The noise component they introduce brings a positive profit to the informed trader, and a negative profit for themselves. The feedback component they introduce increases the variance of the profit.

The first row of the table examines the pure feedback component. It cannot be a separate strategy by itself because uninformed traders have to have the noise component. However, this part can offer important theoretical implications. The results in the first row suggests that pure feedback trading itself does not contribute to the expected profit. It only introduces a variance of profit that grows at the speed of  $\beta^2$ . That is, feedback intensity influences the risk: More aggressive feedback trading (either positive or negative) is associated with higher risks.

With the uninformed traders seen as a whole group, we can compare the second and the third rows of Table 3. The first column suggests that the expected profit (loss) is the same no matter what strategy (i.e., either pure noise (N) or noisy feedback (U)) the uninformed

traders adopt. The second column calculates the risk of each strategy. As long as  $\beta \neq 0$ , noisy-feedback traders' variance of profit is strictly larger than that of the noise traders. So noise trading and feedback trading play very different roles in the market. The level of noise trading shows up in the profit of the informed trader as well as the loss of noisy feedback traders. The greater the noise, the higher the informed trader's profit. Increasing the noise level can increase the variance of noisy feedback traders' profit. However, because the uninformed traders' expected loss is even larger with higher  $\sigma$ , noise trading does not change the risk-adjusted profit (as can be seen from row 2 column 3 of Table 3). For noisy feedback traders in row 3, increasing  $\beta$  can increase the variance of the profit without changing the expected profit. In other words, noisy feedback traders as a whole do bear higher risk than noise traders. Given a negative expected profit, increasing the variance provides an opportunity to increase the chance of getting a positive profit, but at the cost of additional risk without increasing expected profit.

Since Sharpe Ratio is usually positive in comparing the returns of assets, the negative expected profit of uninformed trading in our case gives an illusion of benefit from feedback trading.

Table 3 shows that, for all  $\beta \neq 0$  and  $\sigma$ ,

$$-\frac{1}{\sqrt{\frac{\sigma_v^2}{\sigma^2}\beta^2 + \frac{1}{2}}} > -\frac{1}{\sqrt{\frac{1}{2}}}.$$

Adopting the classical interpretation that higher risk-adjusted return is strictly better, an uninformed trader would always want to increase the feedback. By doing so, the trader has a higher chance to get positive profit, at the cost of the chance to lose much more. Since the expected profit is negative, and the expected profit is solely determined by the noise component, more aggressive trading would not change the expected profit at all. The increased variance creates higher risks without any benefits of improving the returns. Due to this illusion, feedback trading should be implemented with some stop-loss strategies to

manage and contain the risk to a certain level.

## Does the Internet Level the Playing Field?

In our baseline-model setup, the uninformed traders have no information about the real value of the risky asset,  $\tilde{v}$ . Kyle (1985) treats them as noise traders and we examine them as feedback traders in the previous sections. In this section, we extend our model and study the case when the uninformed traders have some noisy signals of the true value.<sup>24</sup> Since the focus of this study is on the impact of Internet-facilitated feedback trading, we only need to examine if this change affects any results related to feedback trading.

Now suppose some of the uninformed traders can obtain the liquidity value with an error, their signal can be described as  $\tilde{v} + \varepsilon$ , where the additional term  $\varepsilon$  denotes the deviation of their signal from the true liquidation value. We assume  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ ,  $\sigma_\varepsilon^2 > 0$ , and  $\varepsilon$  is independent from  $\tilde{v}$ . The informed trader, the market makers and the partially informed traders all know  $\sigma_\varepsilon$ . Nobody knows  $\varepsilon$ . As a result, the informed trader cannot take any actions with respect to  $\varepsilon$ . The partially informed trader certainly does not know  $\varepsilon$ , otherwise, it would be easy to infer  $\tilde{v}$ . The noise traders remain to know nothing about  $\tilde{v}$  or  $\varepsilon$ . The partially informed traders can only observe  $\tilde{v} + \varepsilon$  and these two terms cannot be separately identified. The demand of the uninformed traders can be described as

$$dX_U(t) = \gamma_t(\tilde{v} + \varepsilon - P_t)dt + \beta_t dP_t + \sigma dW_t, \quad (21)$$

where  $\gamma_t$  is to be endogenously decided in the equilibrium. There are three components in the uninformed traders' demand: the noisy-signal part, the feedback part, and the noise trading part. The noisy-signal part is very similar to the one for the informed trader. Now the market can be described more as follows. There is one informed trader as described

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<sup>24</sup>We are indebted to the Associate Editor for suggesting this extension.

before. This informed trader and the market makers further know  $\sigma_\varepsilon$  and do not know the exact value of  $\varepsilon$ . There is a group of uninformed traders, some of whom know the value of  $\tilde{v} + \varepsilon$ . The aggregate demand is given by (21). The informed trader's maximization problem searches for an optimal  $\alpha_t$ . At the same time, the uninformed traders with imprecise signals search for an optimal  $\gamma_t$  to maximize their profits.

This formulation is consistent with similar models in the literature that specifically consider competition between informed traders (Back et al. 2000; Mendelson and Tunca 2003; Pasquariello 2006). We use the following theorem to show that all results related to feedback trading remain unchanged even if the uninformed traders have some noisy signals of the liquidation value.

**Theorem 7.** *Suppose some of the uninformed traders have some noisy signals of the liquidation value and the uninformed traders' orders can be described by Equation (21), then (1) the informed trader's optimal strategy,  $\alpha_t^*$ , is not affected by feedback trading, (2) the uninformed traders' optimal strategy in leveraging their signals,  $\gamma_t^*$ , is independent from feedback trading, and (3) market depth remains a linear function of feedback.*

Like in prior studies of competition among informed traders (e.g., Back et al. 2000, Mendelson and Tunca 2003, among others), competition between the informed trader and the partially informed traders will increase the speed of information getting released. The informed trader would then strengthen the strategy  $\alpha_t^*$  when there is competition (i.e.,  $\alpha_t^* > \alpha_t$ ). Given the assumption  $\alpha_t > \beta_t$  required for establishing stable equilibrium, the technical assumption that  $\alpha_t^* > \beta_t$  is automatically satisfied.

By “market depth remains a linear function of feedback” we mean that  $1/\lambda_t$  is a linear function of  $\beta_t$ . This can be seen from the “Proof of Theorem 7” in Appendix C. In equation (35), market depth ( $1/\lambda_t$ ) can be written as

$$\frac{1}{\lambda_t} = \frac{\sigma + \beta_t (\alpha_t^* \delta_{11}(t) + \gamma_t^* \delta_{12}(t))}{\alpha_t^* \delta_{11}(t) + \gamma_t^* \delta_{12}(t)} = \beta_t + \frac{\sigma}{\alpha_t^* \delta_{11}(t) + \gamma_t^* \delta_{12}(t)}.$$

From the derivation of  $\delta_{11}(t)$  and  $\delta_{12}(t)$  we know that  $\beta_t$  does not appear in them. Similarly,  $\beta_t$  does not appear in other variables such as  $\alpha_t^*$  and  $\gamma_t^*$ . Therefore, market depth is linear in feedback. Since market depth is a measure of market liquidity, the linearity result establishes important implications on feedback trading's effect on liquidity. This result is intuitive because feedback trading does not contain any new information. Although the competition is tightened when the uninformed traders possess imprecise signals of the liquidation value, feedback trading's impact would not change.

## Concluding Remarks

Information plays the most critical role in the financial market; and information technology has been transforming this market for decades. Robert McNamara, former Ford Motors CEO and once president of the World Bank, famously said "A computer does not substitute for judgment any more than a pencil substitutes for literacy. But writing without a pencil is no particular advantage." Despite the promise of the use of IT in the financial market, little research has been done about how the use of IT can affect the market in the IT-Finance literature. Understanding the new technological tools, and the people who use them, provides insights into the market itself. Once we have at our disposal technological innovations in the financial market, there is no need to belabor the importance of characterizing the changes brought about by these innovations.

## Major Findings

In this paper, we study the implication of increased feedback trading as a result of increased Internet trading in the financial market. We develop an equilibrium model of trading in which uninformed traders utilize online trading tools to implement feedback strategies. Uninformed traders play various strategies according to their observations of the price process.

In aggregate, uninformed traders create a feedback effect that is *ex ante* unpredictable.

We show that, once uninformed traders are empowered by online trading tools and use past prices as a source of information, (1) strategic online traders who adopt more aggressive feedback strategies as a whole do not outperform less aggressive ones in terms of the expected profit they earn; (2) feedback trading does not affect market price process; (3) an informed trader's equilibrium strategy and expected profit are not affected by feedback trading; (4) if uninformed traders rely on traditional risk-adjusted measures of profit such as the Shape Ratio, they would increase their feedback trading intensity, but doing so will create much higher risk without increasing (reducing) their expected profit (loss); and (5) the presence of feedback trading in the market affects the sensitivity of the market price to changes in demand, but the speed at which information gets incorporated into price remains unchanged. These results are largely consistent with empirical findings (Barber and Odean 2002; Choi et al. 2002).

## **Theoretical Implications**

This paper contributes to IS-Finance research by deepening our understanding of the impact of IT on the financial market. Most prior studies in this literature focus mainly on the brokerage side of the market to study incentives and competitions in adopting electronic trading systems. Without looking at the investor side, however, it is difficult to assess the impact of IT on investors, arguably the most important participants in the market. This study fills such a gap and examines the consequences of the introduction of the trading tools. It taps into IT's role of aggregating and presenting information, one dimension of the IS discipline's core properties (Benbasat and Zmud 2003), and we answer a call for research bringing new theoretical lens to the IS area (Venkatesh et al. 2007).

In building the theory, this paper adopts a well-established theoretical framework from the finance literature and extends it to accommodate the research questions in the IS context.



It demonstrates the complementarity of IS research with other business disciplines. Although feedback strategies are relatively common in markets, “academic research has until recently hardly addressed the feedback model” (Shiller 2003). While most previous explanations for the existence of feedback trading are based on arguments of systematic biases in human judgments, our study gives a theoretical examination of Internet-facilitated trading with the rational-expectations framework.

## **Managerial Implications**

Compared to earlier studies in the literature, the research topic is substantially different. While earlier studies examine the incentives behind firm and individual adoptions of electronic trading systems, this study moves one step further to study the impact of such adoptions. With the development of business use of IT and the realization of IT value, more research questions will emerge. In this sense, this study only touches the tip of the iceberg of many potential research questions in this field.

When new technologies appear, there are typically two types of reactions. One natural inclination is to applaud the potential in increased efficiency and reduced cost. The other is a fear about the mis-use of the new technology. This study offers some managerial implications about whether the implementation and use of online trading tools are to be applauded or feared.

For regulators of the financial market, our results echo Arthur Levitt’s concern on investing without assessing specific goals and risk tolerances. When investors adopt traditional risk-adjusted profit measures such as Shape Ratio, feedback trading gives them a chance to increase such profit measures. However our calculation shows that this is merely an illusion because their increased risk does not get compensated by higher expected profit. When there are budget constraints, these traders can quickly run out of budget and suffer significant losses. Since feedback trading has limited impact on the informed trader and the price

process. Our results also suggest that uninformed traders' feedback trading does not pose significant costs to other market participants. The influx of inexperienced feedback traders as a result of the ease and speed of the Internet should not be worried too much if market stability is a major concern.

## **Limitations and Future Research**

There are a number of limitations in our framework. Like in other studies that rely on a rational-expectations framework, the assumption of semi-strong market efficiency can still be too strong in reality. What happens if the informed traders are not powerful enough to bring price back to reflect the fundamentals? If this is the case, then the noise-trader approach as a theoretical foundation should be modified. We believe our model related to feedback trading can be generalized when such extensions about noise trading is available. One key take away from our analysis suggests that noise trading determines the profitability (or losses) of traders, and feedback trading determines the variance of such expected profits. This result is likely to remain true in alternative models of noise trading.

In this paper, we only consider one informed trader. Several studies based on the Kyle (1985) framework focus on competition amongst informed traders (Back et al. 2000; Mendelson and Tunca 2003; Pasquariello 2006). The key insight obtained in these studies is that with competition, informed traders release their information more quickly. We consider one variant of this problem in our extension when we relax the assumption that the uninformed traders know nothing about the liquidation value of the asset. This extension can be considered as a model of competition between two types of informed traders, one with accurate information and one with only noisy signals. We show that our results related to feedback trading remain qualitatively unchanged even with informed traders' competition. We are optimistic about the generalizability of our results related to feedback trading in other market situations. However, additional research will need to be done in order to fully understand

how competition affects the impact of feedback trading.

In reality, uninformed online traders can choose strategies that do not rely on past prices. A more elaborate model may relax this assumption. Our model only touches upon the simplest possible form of strategies used by these traders. We believe a useful extension to this work would be to consider the interactions between the informed trader and such uninformed traders who adopt more sophisticated strategies.

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## Appendix A

**Proof of Proposition 1:** We first show that for all  $t$ ,  $\frac{1}{\lambda_t} \neq \beta_t$ . Plugging Equations (3) and (6) into (7) gives

$$dP_t = \lambda_t \beta_t dP_t + \lambda_t \sigma dW_t + \lambda_t \alpha_t (\tilde{v} - P_t) dt.$$

If  $\frac{1}{\lambda_t} = \beta_t$ , then  $\sigma dW_t = -\alpha_t (\tilde{v} - P_t) dt$  holds for all  $\sigma > 0$  and  $\alpha_t > 0$ . Mathematically, it incorrectly implies that the Brownian motion is determined by a drift in time. From a practical point of view, it incorrectly implies that informed traders bring only noise into the market.

When  $\frac{1}{\lambda_t} \neq \beta_t$ , we have

$$dP_t = \frac{\lambda_t \alpha_t}{1 - \lambda_t \beta_t} (\tilde{v} - P_t) dt + \frac{\lambda_t \sigma}{1 - \lambda_t \beta_t} dW_t. \quad (22)$$

Note that (22) is under filtration of  $\mathcal{F}'_t = \mathcal{F}_t \vee \sigma(\tilde{v})$ . For a given  $\mathcal{F}_t$ , taking the conditional expectation of (22) yields

$$dP_t = \frac{\lambda_t \sigma}{1 - \lambda_t \beta_t} dW_t. \quad (23)$$

This is a stochastic differential equation of  $P_t$  under filtration  $\mathcal{F}_t$ . To examine the properties of the price process, we need to apply the filtering lemma by Lipster and Shiryaev (1977), which helps answer the following question: Given the observations of the stochastic process  $P_t$ , what is the best estimate of the state  $\tilde{v}$  based on these observations?

First, let  $V_t = E(\tilde{v} | \mathcal{F}_t)$ , and consider the filtering of  $\tilde{v}$  with respect to  $\{\mathcal{F}_t\}_{0 \leq t \leq 1}$ . By the filtering lemma, we have

$$dV_t = \delta(t) \frac{\lambda_t \alpha_t}{1 - \lambda_t \beta_t} \left( \frac{1 - \lambda_t \beta_t}{\lambda_t \sigma} \right)^2 \frac{\lambda_t \sigma}{1 - \lambda_t \beta_t} dW_t,$$

thus

$$dV_t = \delta(t) \frac{\alpha_t}{\sigma} dW_t, \quad (24)$$

where

$$\delta(t) \equiv E\left[(\tilde{v} - V_t)^2 | \mathcal{F}_t\right]$$

satisfies the following one-dimensional Riccati differential equation:

$$\frac{d\delta(t)}{dt} = -\delta(t) \left( \frac{\lambda_t \alpha_t}{1 - \lambda_t \beta_t} \right)^2 \left( \frac{1 - \lambda_t \beta_t}{\lambda_t \sigma} \right)^2 \delta(t).$$

That is,

$$\frac{d\delta(t)}{dt} = -\frac{\alpha_t^2}{\sigma^2} \delta^2(t)$$

with initial value

$$\delta(0) = \sigma_v^2.$$

The solution to this equation is

$$\delta(t) = \left[ \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right]^{-1}. \quad (25)$$

Plugging (25) into (24) and using the semi-strong efficiency condition gives

$$dP_t = \left[ \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right]^{-1} \frac{\alpha_t}{\sigma} dW_t. \quad (26)$$

Comparing coefficients of (23) and (26) yields

$$\left[ \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right]^{-1} \frac{\alpha_t}{\sigma} = \frac{\lambda_t \sigma}{1 - \lambda_t \beta_t}. \quad (27)$$

Thus

$$\frac{\sigma^2}{\alpha_t} \left[ \sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds \right] = \frac{1}{\lambda_t} - \beta_t. \quad (28)$$

Since  $\alpha_t$  is strictly positive, we can see that when the market is semi-strong efficient the depth of the market  $\frac{1}{\lambda_t}$  is always greater than  $\beta_t$ .

Equation (27) can be rewritten as

$$\frac{\left(\frac{\alpha_t}{\sigma}\right)^2}{\left[\sigma_v^{-2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds\right]^2} = \left(\frac{\lambda_t \sigma}{1 - \lambda_t \beta_t}\right)^2.$$

Integrating the above equation with respect to  $dt$  yields

$$\alpha_t = \frac{\lambda_t \sigma^2}{\left(\sigma_v^2 - \int_0^t \left(\frac{\lambda_s \sigma}{1 - \lambda_s \beta_s}\right)^2 ds\right) (1 - \lambda_t \beta_t)}. \quad (29)$$

Again, since  $\alpha_t$  is strictly positive, it is easy to see that for all  $t \in [0, 1]$

$$\sigma_v^2 - \int_0^t \left(\frac{\lambda_s \sigma}{1 - \lambda_s \beta_s}\right)^2 ds > 0.$$

*Q.E.D.*

**Proof of Proposition 2:** By Schwartz inequality and the constraint, we know that

$$\int_0^1 \frac{\lambda_s}{1 - \lambda_s \beta_s} ds \leq \left( \int_0^1 \left[ \frac{\lambda_s}{1 - \lambda_s \beta_s} \right]^2 ds \right)^{\frac{1}{2}} \leq \frac{\sigma_v}{\sigma}.$$

The equality holds if and only if for any  $t \in [0, 1]$ ,

$$\frac{\lambda_t}{1 - \lambda_t \beta_t} = \frac{\sigma_v}{\sigma}. \quad (30)$$

*Q.E.D.*

**Proof of Theorem 3:** Equation (16) can be obtained directly from (30). (17) is obtained by plugging (16) into (29). (27) and (25) combined yields (18). Finally, (19) is obtained by combining (15) and (30).

*Q.E.D.*

**Proof of Proposition 4:** Under the assumptions in our model, the profit earned by uninformed traders can be expressed by

$$E \left[ (1-) \int_0^t (\tilde{v} - P_t) dX_U(t) \right].$$

And,

$$\begin{aligned} & E \left[ (1-) \int_0^t (\tilde{v} - P_t) dX_U(t) \right] \\ &= E \left[ (1-) \int_0^t (\tilde{v} - P_t) (\beta_t dP_t + \sigma dW_t) \right] \\ &= E \left[ (1-) \int_0^t (\tilde{v} - P_t) \left( \frac{\beta_t \lambda_t \alpha_t}{1 - \lambda_t \beta_t} (\tilde{v} - P_t) dt + \left( \sigma + \frac{\lambda_t \beta_t \sigma}{1 - \lambda_t \beta_t} \right) dW_t \right) \right] \\ &= E \left[ (1-) \int_0^t \frac{\beta_t \lambda_t \alpha_t}{1 - \lambda_t \beta_t} (\tilde{v} - P_t)^2 dt \right] + E \left[ (1-) \int_0^t (\tilde{v} - P_t) \frac{\sigma}{1 - \lambda_t \beta_t} dW_t \right] \\ &= \int_0^t \frac{\beta_t \lambda_t \alpha_t}{1 - \lambda_t \beta_t} \delta(t) dt + E \left[ (1-) \int_0^t (\tilde{v} - (P_0 + \int_0^t \lambda_s dX_I(s) + \int_0^t \lambda_s dX_U(t))) \frac{\sigma}{1 - \lambda_t \beta_t} dW_t \right] \\ &= \sigma_v^2 \int_0^t \beta_t dt - E \left[ (1-) \int_0^t \left( \int_0^t \lambda_s dX_U(t) \right) \frac{\sigma}{1 - \lambda_t \beta_t} dW_t \right] \\ &= \sigma_v^2 \int_0^t \beta_t dt - E \left[ (1-) \int_0^t \left( \int_0^t \frac{\beta_s \lambda_s^2 \alpha_s}{1 - \lambda_s \beta_s} (\tilde{v} - P_s) ds + \frac{\lambda_s \sigma}{1 - \lambda_s \beta_s} dW_s \right) \frac{\sigma}{1 - \lambda_t \beta_t} dW_t \right] \\ &= \sigma_v^2 \int_0^t \beta_t dt - E \left[ (1-) \int_0^t \frac{\sigma_v \sigma}{1 - \lambda_t \beta_t} W_t dW_t \right] \\ &= \sigma_v^2 \int_0^t \beta_t dt - \sigma_v^2 \int_0^t \left( \frac{\sigma}{\sigma_v} + \beta_t \right) dt \\ &= -\sigma_v \int_0^t \sigma dt \\ &= -\sigma_v \sigma t \end{aligned}$$

The result is obtained from (7), (22), (16), (18), and the transformation relation between the Itô and the  $(1-)$  stochastic integration. The last equation assumes that  $\sigma$  is not a function of  $t$ . If  $\sigma$  is indeed a function of  $t$ , the result is not changed:  $\int_0^t \sigma dt$  simply measures the average variance of noise up to time  $t$ . Whether  $\sigma$  is a function of time does not change any of our results.

*Q.E.D.*

**Proof of Theorem 5:** Without loss of generality, we suppose  $P_0 = 0$ . The second moment of the

informed trader's profits is

$$\begin{aligned}
& E \left[ (1-t) \int_0^1 (\tilde{v} - P_t) dX_I(t) \right]^2 = E \left[ (1-t) \int_0^1 (\tilde{v} - P_t)^2 \cdot \frac{\sigma}{\sigma_v} \cdot \frac{1}{1-t} dt \right]^2 \\
& = E \left[ \int_0^1 \left( \tilde{v} - \tilde{v}t - (1-t) \int_0^t \frac{\sigma_v}{1-s} dW_s \right)^2 \cdot \frac{\sigma}{\sigma_v} \cdot \frac{1}{1-t} dt \right]^2 \\
& = E \left[ \int_0^1 (1-t) \left( \tilde{v} - \int_0^t \frac{\sigma_v}{1-s} dW_s \right)^2 \cdot \frac{\sigma}{\sigma_v} dt \right]^2 \\
& = E \left[ \int_0^1 (1-t) \left( \tilde{v}^2 - 2\tilde{v} \int_0^t \frac{\sigma_v}{1-s} dW_s + \left( \int_0^t \frac{\sigma_v}{1-s} dW_s \right)^2 \right) \cdot \frac{\sigma}{\sigma_v} dt \right]^2 \\
& = E \left[ \int_0^1 (1-t) \tilde{v}^2 \frac{\sigma}{\sigma_v} dt - 2\tilde{v} \int_0^1 (1-t) \frac{\sigma}{\sigma_v} \left( \int_0^t \frac{\sigma_v}{1-s} dW_s \right) dt \right. \\
& \quad \left. + \int_0^1 (1-t) \left( \int_0^t \frac{\sigma_v}{1-s} dW_s \right)^2 \cdot \frac{\sigma}{\sigma_v} dt \right]^2
\end{aligned}$$

Define the first term by  $A_1$ ,

$$A_1 \equiv \int_0^1 (1-t) \tilde{v}^2 \frac{\sigma}{\sigma_v} dt = \frac{\sigma}{2\sigma_v} \tilde{v}^2.$$

Integrating by parts (stochastic integration, generalized Itô formula), we can have

$$\int_0^t \frac{1}{1-s} dW_s = \frac{W_s}{1-s} \Big|_0^t - \int_0^t W_s d\left(\frac{1}{1-s}\right) = \frac{W_t}{1-t} - \int_0^t \frac{W_s}{(1-s)^2} ds$$

By interchangeability of ordinary Riemann integration, we can calculate

$$\begin{aligned}
A_2 & \equiv \int_0^1 (1-t) 2\tilde{v} \left( \int_0^t \frac{\sigma_v}{1-s} dW_s \right) \frac{\sigma}{\sigma_v} dt \\
& = 2\tilde{v}\sigma \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right) dt \\
& = 2\tilde{v}\sigma \int_0^1 (1-t) \left( \frac{W_t}{1-t} - \int_0^t \frac{W_s}{(1-s)^2} ds \right) dt \\
& = 2\tilde{v}\sigma \left[ \int_0^1 W_t dt - \int_0^1 \int_0^t (1-t) \frac{W_s}{(1-s)^2} ds dt \right] \\
& = 2\tilde{v}\sigma \left[ \int_0^1 W_t \left( 1 - \frac{1}{1-t} + \frac{1}{2} \cdot \frac{1+t}{1-t} \right) dt \right] \\
& = \tilde{v}\sigma \int_0^1 W_t dt.
\end{aligned}$$

And the last term

$$\begin{aligned} A_3 &\equiv \int_0^1 (1-t) \cdot \frac{\sigma}{\sigma_v} \left( \int_0^t \frac{\sigma_v}{1-s} dW_s \right)^2 dt \\ &= \sigma \sigma_v \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \end{aligned}$$

The informed trader's variance of the profit is

$$\begin{aligned} &E[A_1 + A_2 + A_3]^2 - (E[\tilde{\pi}(1)])^2 \\ &= E[A_1^2 + A_2^2 + A_3^2 + 2A_1A_2 + 2A_1A_3 + 2A_2A_3] - (E[\tilde{\pi}(1)])^2 \\ &= \frac{\sigma^2}{4\sigma_v^2} E[\tilde{v}^4] + \sigma^2 E[\tilde{v}^2] E \left[ \int_0^1 W_t dt \right]^2 + \sigma_v^2 \sigma^2 E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \right]^2 \\ &\quad + \frac{1}{2} \sigma^2 E[\tilde{v}^2] - \sigma^2 \sigma_v^2 \\ &= \sigma_v^2 \sigma^2 \left[ \frac{1}{4} + E \left[ \int_0^1 W_t dt \right]^2 + E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \right]^2 + \frac{1}{2} - 1 \right] \\ &= \frac{1}{4} \sigma_v^2 \sigma^2. \end{aligned}$$

We have used the assumption that  $\tilde{v}$  is independent of the Brownian motion  $W_t$ , and the expectation of  $\tilde{v}$  is zero, i.e.,  $\bar{P} \equiv E[\tilde{v}] = 0$ , and the last equality is obtained from results in Appendix B.

We continue to calculate the variance of the uninformed traders' profits. For simplicity, we suppose that  $\beta_t$  is a constant over  $t$ , denoted by  $\beta$ , the second moment of the uninformed traders' profits is

$$\begin{aligned} &E \left[ (1-) \int_0^1 (\tilde{v} - P_t) dX_U(t) \right]^2 \\ &= E \left[ (1-) \int_0^1 (\tilde{v} - P_t) \cdot (\beta dP_t + \sigma dW_t) \right]^2 \\ &= E \left[ (1-) \int_0^1 (\tilde{v} - P_t) \cdot \beta dP_t + (1-) \int_0^1 (\tilde{v} - P_t) \sigma dW_t \right]^2 \\ &= E \left[ (1-) \int_0^1 (\tilde{v} - P_t)^2 \beta \cdot \frac{1}{1-t} dt + (1-) \int_0^1 (\tilde{v} - P_t) (\beta \sigma_v + \sigma) dW_t \right]^2 \\ &= E \left[ \int_0^1 \frac{\beta}{1-t} (\tilde{v} - P_t)^2 dt \right]^2 + 2E \left[ \left( \int_0^1 \frac{\beta}{1-t} (\tilde{v} - P_t)^2 dt \right) (1-) \int_0^1 (\tilde{v} - P_t) (\beta \sigma_v + \sigma) dW_t \right] \\ &\quad + E \left[ (1-) \int_0^1 (\tilde{v} - P_t) (\beta \sigma_v + \sigma) dW_t \right]^2 \end{aligned}$$

The first term

$$\begin{aligned}
B_1 &\equiv E \left[ \beta^2 \left( \int_0^1 \frac{1}{1-t} (\tilde{v} - P_t)^2 dt \right)^2 \right] \\
&= \beta^2 \cdot \frac{\sigma_v^2}{\sigma^2} \sigma_v^2 \sigma^2 \left[ \frac{5}{4} + E \left[ \int_0^1 W_t dt \right]^2 + E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \right]^2 \right] \\
&= \beta^2 \sigma_v^4 \left[ \frac{5}{4} + E \left[ \int_0^1 W_t dt \right]^2 + E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \right]^2 \right] \\
&= \frac{7}{4} \beta^2 \sigma_v^4
\end{aligned}$$

The last equality is obtained from Appendix B.

The second term

$$\begin{aligned}
B_2 &\equiv 2\beta(\beta\sigma_v + \sigma)E \left[ \left( \int_0^1 \frac{1}{1-t} (\tilde{v} - P_t)^2 dt \right) (1-) \int_0^1 (\tilde{v} - P_t) dW_t \right] \\
&= 2\beta(\beta\sigma_v + \sigma) \frac{\sigma_v}{\sigma} E \left[ (A_1 + A_2 + A_3) \left( \int_0^1 (\tilde{v} - P_t) dW_t + \int_0^1 -\sigma_v dt \right) \right] \\
&= 2\beta(\beta\sigma_v + \sigma) \frac{\sigma_v}{\sigma} E \left[ (A_1 + A_2 + A_3) \left[ \int_0^1 (1-t) \left( \tilde{v} - \int_0^t \frac{\sigma_v}{1-s} dW_s \right) dW_t - \sigma_v \right] \right] \\
&= 2\beta(\beta\sigma_v + \sigma) \frac{\sigma_v}{\sigma} \left[ E \left( (A_1 + A_2 + A_3) \tilde{v} \int_0^1 (1-t) dW_t \right) \right. \\
&\quad \left. - E \left( (A_1 + A_2 + A_3) \int_0^1 (1-t) \int_0^t \frac{\sigma_v}{1-s} dW_s dW_t \right) - E(A_1 + A_2 + A_3) \sigma_v \right] \\
&= 2\beta(\beta\sigma_v + \sigma) \frac{\sigma_v}{\sigma} \left[ \sigma_v^2 \sigma E \left( \int_0^1 W_t dt \right)^2 \right. \\
&\quad \left. - \sigma_v E \left( (A_1 + A_2 + A_3) \int_0^1 (1-t) \int_0^t \frac{1}{1-s} dW_s dW_t - 1 \right) \right] \\
&= 2\beta(\beta\sigma_v + \sigma) \frac{\sigma_v}{\sigma} \left[ \sigma_v^2 \sigma E \left( \int_0^1 W_t dt \right)^2 \right. \\
&\quad \left. - \sigma \sigma_v^2 E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \cdot \int_0^1 (1-t) \int_0^t \frac{1}{1-s} dW_s dW_t \right] - \frac{7}{4} \sigma \sigma_v^2 \right] \\
&= 2\beta \sigma_v^3 (\beta\sigma_v + \sigma) \left[ E \left( \int_0^1 W_t dt \right)^2 \right. \\
&\quad \left. - E \left( \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \cdot \int_0^1 (1-t) \int_0^t \frac{1}{1-s} dW_s dW_t \right) - \frac{7}{4} \right] \\
&= -3\beta \sigma_v^3 (\beta\sigma_v + \sigma)
\end{aligned}$$

The third term

$$\begin{aligned}
B_3 &\equiv (\beta\sigma_v + \sigma)^2 E \left[ \left( (1-) \int_0^1 (\tilde{v} - P_t) dW_t \right)^2 \right] \\
&= (\beta\sigma_v + \sigma)^2 E \left[ \left( \int_0^1 (\tilde{v} - P_t) dW_t + \int_0^1 -\sigma_v dt \right)^2 \right] \\
&= (\beta\sigma_v + \sigma)^2 \left[ \int_0^1 E(\tilde{v} - P_t)^2 dt + \sigma_v^2 \right] \\
&= \frac{3}{2} \sigma_v^2 (\beta\sigma_v + \sigma)^2
\end{aligned}$$

Here we have used the isometry property of the stochastic integral. The uninformed traders' variance of profits is therefore

$$\begin{aligned}
&B_1 + B_2 + B_3 - (\sigma^2 \sigma_v^2) \\
&= \frac{7}{4} \beta^2 \sigma_v^4 - 3\beta\sigma_v^3 (\beta\sigma_v + \sigma) + \frac{3}{2} \sigma_v^2 (\beta\sigma_v + \sigma)^2 - (\sigma^2 \sigma_v^2) \\
&= \sigma_v^4 \beta^2 + \frac{1}{2} \sigma^2 \sigma_v^2
\end{aligned}$$

*Q.E.D.*

## Appendix B

Here we show how to calculate some expectations useful for Appendix A. First, we calculate  $E \left[ \int_0^1 W_t dt \right]^2$ :

$$\begin{aligned}
E \left[ \int_0^1 W_t dt \right]^2 &= E \left[ \int_0^1 W_t dt \int_0^1 W_s ds \right] \\
&= \int_0^1 \int_0^1 E[W_t W_s] dt ds \\
&= \int_0^1 \int_0^1 \min(s, t) dt ds \\
&= \int_0^1 \left( \int_0^s t dt + \int_s^1 s dt \right) ds \\
&= \int_0^1 \left( \frac{s^2}{2} + s(1-s) \right) ds = \frac{1}{3}
\end{aligned}$$

In the following, we calculate

$$\Gamma \equiv E \left[ \int_0^1 (1-t) \left( \int_0^t \frac{1}{1-s} dW_s \right)^2 dt \cdot \int_0^1 (1-t) \int_0^t \frac{1}{1-s} dW_s dW_t \right].$$



Let

$$\begin{aligned} I_t &= \int_0^t \frac{1}{1-s} dW_s \\ X_\tau &= \int_0^\tau (1-t) I_t^2 dt \\ Y_\tau &= \int_0^\tau (1-t) I_t dW_t, \end{aligned}$$

where  $I_t$  and  $Y_\tau$  are martingales. What we want is

$$X_1 Y_1.$$

Integrating by parts,

$$X_1 Y_1 = \int_0^1 X_\tau dY_\tau + \int_0^1 Y_\tau dX_\tau.$$

Since  $Y_\tau$  is a martingale,

$$\begin{aligned} E[X_1 Y_1] &= E \left[ \int_0^1 Y_\tau dX_\tau \right] \\ &= E \left[ \int_0^1 Y_\tau (1-\tau) I_\tau^2 d\tau \right] \\ &= \int_0^1 (1-\tau) E[Y_\tau I_\tau^2] d\tau. \end{aligned}$$

Integrating by parts, we have

$$Y_\tau I_\tau^2 = \int_0^\tau I_\tau^2 dY_\tau + \int_0^\tau Y_\tau dI_\tau^2 + \frac{1}{2} \int_0^\tau d \langle Y, I^2 \rangle_t,$$

where  $\langle X, Y \rangle_t$  denotes the quadratic variation process of  $X$  and  $Y$ .

$$dI_t^2 = 2I_t dI_t + d \langle I, I \rangle_t = 2I_t dI_t + \frac{1}{(1-t)^2} dt,$$

therefore

$$\begin{aligned} E[Y_\tau I_\tau^2] &= E \left[ \int_0^\tau Y_t \frac{1}{(1-t)^2} dt + \frac{1}{2} \int_0^\tau (1-t) I_t \cdot 2I_t \frac{1}{1-t} dt \right] \\ &= \int_0^\tau E[I_t^2] dt = \int_0^\tau \left( \int_0^t \frac{1}{(1-s)^2} ds \right) dt \\ &= \int_0^\tau \left( \frac{1}{1-t} - 1 \right) dt = -\ln(1-\tau) - \tau. \end{aligned}$$

$$\begin{aligned}
E[X_1 Y_1] &= \int_0^1 (1-\tau) E[Y_\tau I_\tau^2] d\tau \\
&= - \int_0^1 x \ln(x) dx - \int_0^1 (1-\tau)\tau d\tau = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}.
\end{aligned}$$

Hence, we obtain

$$\Gamma = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}.$$

Next, we calculate  $E\left[\int_0^1 (1-t) \left(\int_0^t \frac{1}{1-s} dW_s\right)^2 dt\right]^2$ . Using the same notation as above, what we want is  $E[X_1^2]$ . Integrating by parts,

$$X_1 X_1 = 2 \int_0^1 X_\tau dX_\tau.$$

Hence

$$\begin{aligned}
E[X_1^2] &= 2E\left[\int_0^1 X_\tau dX_\tau\right] \\
&= 2E\left[\int_0^1 X_\tau (1-\tau) I_\tau^2 d\tau\right] \\
&= 2 \int_0^1 (1-\tau) E[X_\tau I_\tau^2] d\tau.
\end{aligned}$$

Integrating by parts, we have

$$\begin{aligned}
X_\tau I_\tau^2 &= \int_0^\tau I_t^2 dX_t + \int_0^\tau X_t dI_t^2 + \frac{1}{2} \int_0^\tau d\langle X, I^2 \rangle_t \\
&= \int_0^\tau (1-t) I_t^4 dt + \int_0^\tau X_t dI_t^2,
\end{aligned}$$

where  $\langle X, Y \rangle_t$  denotes the quadratic variation process of  $X$  and  $Y$ .

$$dI_t^2 = 2I_t dI_t + d\langle I, I \rangle_t = 2I_t dI_t + \frac{1}{(1-t)^2} dt,$$

therefore

$$\begin{aligned}
E[X_\tau I_\tau^2] &= E\left[\int_0^\tau (1-t) I_t^4 dt + \int_0^\tau X_t \frac{1}{(1-t)^2} dt\right] \\
&= \int_0^\tau (1-t) E[I_t^4] dt + \int_0^\tau E[X_t] \frac{1}{(1-t)^2} dt \\
&= \int_0^\tau (1-t) \frac{3t^2}{(1-t)^2} dt + \frac{1}{2} \int_0^\tau \frac{t^2}{(1-t)^2} dt \\
&= -3\left(\tau + \frac{\tau^2}{2}\right) - 3\ln(1-\tau) + \frac{1}{2} \left[\tau + \ln(1-\tau) + \frac{1}{1-\tau} - 1\right] \\
&= -\frac{5}{2}(\tau + \ln(1-\tau)) + \frac{1}{2} \left[\frac{1}{1-\tau} - 1 - 3\tau^2\right].
\end{aligned}$$

Here we have used the result that  $I_t \sim N(0, \frac{t}{1-t})$ , then  $E[I_t^4] = \frac{3t^2}{(1-t)^2}$  and  $E(X_t) = \frac{t^2}{2}$ .

$$\begin{aligned} E[X_1^2] &= 2 \int_0^1 (1-\tau) E[X_\tau I_\tau^2] d\tau \\ &= - \int_0^1 5x \ln(x) dx - 5 \int_0^1 (1-\tau)\tau d\tau + \int_0^1 \tau d\tau - \int_0^1 (1-t)dt - \int_0^1 3\tau^2(1-\tau)d\tau \\ &= \frac{5}{4} - \frac{5}{6} - 1 + \frac{3}{4} = \frac{1}{6}. \end{aligned}$$

## Appendix C

### Proof of Theorem 7:

Inserting the uninformed traders' new demand (21) to the pricing rule (7), we can obtain

$$dP_t = \left[ \frac{\lambda_t \alpha_t}{1 - \lambda_t \beta_t} \tilde{v} + \frac{\lambda_t \gamma_t}{1 - \lambda_t \beta_t} (\tilde{v} + \varepsilon) - \frac{\lambda_t (\alpha_t + \gamma_t)}{1 - \lambda_t \beta_t} P_t \right] dt + \frac{\lambda_t \sigma}{1 - \lambda_t \beta_t} dW_t.$$

The logic of deriving the result is the same as before. However, since we have one additional dimension of uncertainty coming from  $\varepsilon$ , the filtering process needs to deal with the vector  $\begin{pmatrix} \tilde{v} \\ \tilde{v} + \varepsilon \end{pmatrix}$ .

Consequently, the deviation of the price from the liquidation value at time  $t$ ,  $\delta(t)$ , is a matrix

$$\delta(t) \equiv \begin{pmatrix} \delta_{11}(t) & \delta_{12}(t) \\ \delta_{21}(t) & \delta_{22}(t) \end{pmatrix},$$

with  $\delta_{11}(t) = E[\tilde{v} - E(\tilde{v}|\mathcal{F}_t)]^2$ ,  $\delta_{21}(t) = \delta_{12}(t) = E[(\tilde{v} - E(\tilde{v}|\mathcal{F}_t))((\tilde{v} + \varepsilon) - E((\tilde{v} + \varepsilon)|\mathcal{F}_t))]$ , and  $\delta_{22}(t) = E[(\tilde{v} + \varepsilon) - E((\tilde{v} + \varepsilon)|\mathcal{F}_t)]^2$ .

The variance-covariance matrix can be derived as

$$d \begin{pmatrix} \delta_{11}(t) & \delta_{12}(t) \\ \delta_{21}(t) & \delta_{22}(t) \end{pmatrix} = - \begin{pmatrix} \delta_{11}(t) & \delta_{12}(t) \\ \delta_{21}(t) & \delta_{22}(t) \end{pmatrix} \begin{pmatrix} \frac{\alpha_t^2}{\sigma^2} & \frac{\alpha_t \gamma_t}{\sigma^2} \\ \frac{\alpha_t \gamma_t}{\sigma^2} & \frac{\gamma_t^2}{\sigma^2} \end{pmatrix} \begin{pmatrix} \delta_{11}(t) & \delta_{12}(t) \\ \delta_{21}(t) & \delta_{22}(t) \end{pmatrix}' dt, \quad (31)$$

where the symbol  $'$  denotes the transpose of the matrix.

Equation (31) is a matrix Riccati differential equation with initial value

$$\delta(0) = \begin{pmatrix} \delta_{11}(0) & \delta_{12}(0) \\ \delta_{21}(0) & \delta_{22}(0) \end{pmatrix} = \begin{pmatrix} \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

The solution to the equation is

$$\delta(t) = \left[ (\delta(0))^{-1} + \begin{pmatrix} \int_0^t \frac{\alpha_s^2}{\sigma^2} ds & \int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds \\ \int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds & \int_0^t \frac{\gamma_s^2}{\sigma^2} ds \end{pmatrix} \right]^{-1}. \quad (32)$$

After calculation, we obtain that

$$(\delta(0))^{-1} = \begin{pmatrix} \frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} & -\frac{1}{\sigma_\varepsilon^2} \\ -\frac{1}{\sigma_\varepsilon^2} & \frac{1}{\sigma_\varepsilon^2} \end{pmatrix},$$

and

$$\begin{aligned} \delta_{11}(t) &= \frac{\left(\frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\gamma_s^2}{\sigma^2} ds\right)}{\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds\right) \left(\frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\gamma_s^2}{\sigma^2} ds\right) - \left[\int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds - \frac{1}{\sigma_\varepsilon^2}\right]^2}, \\ \delta_{12}(t) = \delta_{21}(t) &= \frac{-\left[\int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds - \frac{1}{\sigma_\varepsilon^2}\right]}{\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds\right) \left(\frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\gamma_s^2}{\sigma^2} ds\right) - \left[\int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds - \frac{1}{\sigma_\varepsilon^2}\right]^2}, \\ \delta_{22}(t) &= \frac{\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds\right)}{\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\alpha_s^2}{\sigma^2} ds\right) \left(\frac{1}{\sigma_\varepsilon^2} + \int_0^t \frac{\gamma_s^2}{\sigma^2} ds\right) - \left[\int_0^t \frac{\alpha_s \gamma_s}{\sigma^2} ds - \frac{1}{\sigma_\varepsilon^2}\right]^2}. \end{aligned}$$

The informed trader's expected profit at time 1 is

$$\begin{aligned} E \left[ \int_0^1 (\tilde{v} - P_t) dX_I(t) \right] &= E \left[ \int_0^1 \alpha_t (\tilde{v} - P_t)^2 dt \right] = \int_0^1 \alpha_t E[(\tilde{v} - P_t)^2] dt \\ &= \int_0^1 \alpha_t \delta_{11}(t) dt. \end{aligned}$$

Same as before, the informed trader chooses  $\alpha_t$  to maximize her expected profit. That is,

$$\alpha_t^* = \arg \max \int_0^1 \alpha_t \delta_{11}(t) dt.$$

Same as  $\delta(t)$  in the baseline model,  $\delta_{11}(t)$  does not involve the feedback parameter  $\beta_t$ . So the informed trader's maximization problem is independent of the feedback intensity.

Similarity, the uninformed trader with imprecise information tries to maximize her profit at time 1. The maximization problem is

$$\gamma_t^* = \arg \max \int_0^1 \gamma_t \delta_{12}(t) dt.$$

Since  $\delta_{12}(t)$  does not involve the feedback parameter  $\beta_t$ , the optimal  $\gamma_t$  is also independent from the feedback intensity.

For the conditional expectation, we have

$$d \begin{pmatrix} E \left[ \tilde{v} | \mathcal{F}_t \right] \\ E \left[ (\tilde{v} + \varepsilon) | \mathcal{F}_t \right] \end{pmatrix} = \delta(t) \begin{pmatrix} \frac{\alpha_t}{\sigma} \\ \frac{\gamma_t}{\sigma} \end{pmatrix} dW_t + \delta(t) \begin{pmatrix} \frac{\alpha_t^2}{\sigma^2} & \frac{\alpha_t \gamma_t}{\sigma^2} \\ \frac{\alpha_t \gamma_t}{\sigma^2} & \frac{\gamma_t^2}{\sigma^2} \end{pmatrix} \begin{pmatrix} \tilde{v} - E[\tilde{v} | \mathcal{F}_t] \\ (\tilde{v} + \varepsilon) - E[(\tilde{v} + \varepsilon) | \mathcal{F}_t] \end{pmatrix} dt, \quad (33)$$

Applying the semi-strong market-efficiency condition  $E[\tilde{v}|\mathcal{F}_t] = P_t$  we have for all  $\alpha_t$  and  $\gamma_t$ ,

$$\frac{\alpha_t}{\sigma}\delta_{11}(t) + \frac{\gamma_t}{\sigma}\delta_{12} = \frac{\lambda_t\sigma}{1 - \lambda_t\beta_t}. \quad (34)$$

Inserting the optimal values  $\alpha_t^*$  and  $\gamma_t^*$ , we get the result about  $\lambda_t$ .

$$\lambda_t = \frac{\alpha_t^*\delta_{11}(t) + \gamma_t^*\delta_{12}(t)}{\sigma + \beta_t(\alpha_t^*\delta_{11}(t) + \gamma_t^*\delta_{12}(t))}. \quad (35)$$

This result is highly consistent with what we have obtained in Theorem (3). The expression of  $\lambda_t$  is very similar to that in the baseline model. The only difference is that the variance of the liquidation value in the baseline model is replaced by the variance and covariance of the liquidation value together with the error.

Overall, this completes the proof that, even if uninformed traders can obtain imprecise signals of information, feedback trading does not affect informed trader's strategy as well as the market price process.

*Q.E.D.*